The Role of Students’ Quantitative Reasoning in Solving Mathematical Problems Based on Cognitive Style

Peran Penalaran Kuantitatif Siswa dalam Memecahkan Masalah Matematika Berdasarkan Gaya Kognitif

Muhammad Muzaini, Muhammad Hasbi*, Nasrun Nasrun

1Universitas Cokroaminoto Palopo
2Institut Agama Islam As’Adiyah Sengkang
3Universitas Muhammadiyah Makassar

*Email Korespondensi: muhammad.hasby80@gmail.com

Abstract

The emphasis of this research was how quantitative reasoning played an active role based on the cognitive style in addressing mathematical problem issues in middle school students. Participants in this study were students of a public middle-school, South Sulawesi, Indonesia. The findings revealed that Field-Independent students may define a problem or circumstance that solves the problem at hand, pick the relation between the quantities, and make quantity and relation-centered calculations. Second, Field-Dependent students use quantitative reasoning to consider the problem context, concentrate on the problem context quantity, deduce the relationship between quantities in the plans produced. The findings of this study are important for developing effective quantitative reasoning as input material for teachers.

Cara merujuk artikel ini:


Abstrak

Penekanan penelitian ini adalah bagaimana penalaran kuantitatif berperan aktif dalam menyikapi masalah matematika berdasarkan gaya kognitif siswa sekolah menengah. Partisipan penelitian ini adalah siswa sekolah menengah negeri, di Sulawesi Selatan, Indonesia. Temuan mengungkapkan bahwa siswa field-independent dapat mendefinisikan masalah atau keadaan dalam memecahkan masalah yang dihadapi, memilih hubungan antara kuantitas, dan membuat kalkulasi yang berpusat pada kuantitas dan hubungan. Kedua, siswa field-dependent menggunakan penalaran kuantitatif untuk mempertimbangkan konteks masalah, berfokus pada kuantitas konteks masalah, menyimpulkan hubungan antara kuantitas dalam rencana yang dihasilkan. Temuan penelitian ini penting untuk mengembangkan penalaran kuantitatif yang efektif sebagai bahan masukan bagi guru.
INTRODUCTION

Mathematical reasoning is a central feature of the topic (Sidenvall et al., 2015). Reasoning is the continuous association of a person's schemes and operations (Ackerman & Thompson, 2017). Students must practice solving routine and non-routine tasks in order to master both procedures and mathematically founded reasoning (Sidenvall et al., 2015). One of the aims of middle-school mathematics is to develop thinking skills. The National Council of Teachers of Mathematics established standards for the mathematics learning process, which must include: problem-solving (Febriyanti & Novitasari, 2019; Hasbi et al., 2019; Juwita & Ariani, 2020; Rahayuningsih et al., 2021), reasoning (Yuwita & Ariani, 2020), and evidence, communication, relationships, and representation, are also important skills to have (NCTM, 2000).

As people deal with certain quantities and quantity relationships, they use their quantitative reasoning (Weber et al., 2014). Quantitative reasoning by Ackerman & Thompson, (2017); Ellis et al. (2019), is an interpretation of a condition into a system in the form of a network of quantities and quantitative relationships. This perspective illustrates that quantitative reasoning covers a broad variety of problems pertaining to quantitative relationships, such as whether students choose to see the additive ratio of two quantities, such as when measuring how big one individual is to another, or multiplicative comparisons, such as calculating how many times one item is greater than another (Ching et al., 2020). Quantitative knowledge is data in the form of numbers and a relationship between them.

For students studying mathematics, quantitative reasoning is a vital mental method (Moore, 2014; Weber et al., 2014). Quantitative reasoning has a wide range of meanings in literature. Discussing mathematical analysis as a skill that all students must learn is one of the objectives of mathematics education (NCTM, 2000). A growing body of research has established quantitative reasoning as an effective way of thinking, which is vital in order to build a productive understanding of a broad range of mathematical principles (Moore, 2014). Various investigators have recognized that quantitative reasoning is essential for the practical learning of mathematical problems by students (Muzaini et al., 2019; Tallman & Frank, 2020).

Tallman & Frank (2020) that quantitative thought is a way of thinking, by definition. There are several statistical thoughts that can be applied productively in order to conceptualize circumstances with regard to volume and quantitative relations. An individual who holds an orientation through a range of mathematical fields to detect and describe observable properties of objects has a cognitive feature of conceptual interpretation and analysis that can be considered quantitative reasoning. The concept of a quantity requires no integer value to be assigned to a specific object attribute. Rather, a measuring method must be taken into account and an adequate unit of measurement must be conceived,
indirectly or directly. Quantification refers to the mental behavior involved in the conception of an appropriate unit of measurement and a measurement procedure, resulting in an interpretation of "what quantity quantities are for, what measures they take, and what measurements they take" (Ackerman & Thompson, 2017).

In addition, quantitative reasoning is a way of defining quantities in a problem, deciding and arranging quantities and linkages between quantities, considering the units of quantities, and performing calculations by referencing quantities in the problem plan or model. According to Moore & Carlson (2012), quantitative reasoning involves behavioral behaviors such as comprehending the problem's meaning, increasing the number of conditions known, assessing the relationship between quantities, and evaluating shifts in quantity. When students argue quantitatively, they construct quantities in the sense of a problem. Quantity is a function of a measurable entity or phenomenon (Weber et al., 2014). The measuring act thus renders the properties of a measuring item or phenomenon into a quantity. Mathematical and mathematical reasoning are two concepts that can’t be separated.

The main objective of the world's mathematics teachers is to combine student thinking with everyday issues, which can be mathematically solved (Mkhatshwa, 2020). This goal can be achieved by enhancing students’ mathematical attitudes. This will be improved if students are able to employ quantitative thinking. The study results indicate that quantitative reasoning has a significant impact on students, since it makes students understand and argue well that quantitative facts are based upon and these argumentations can be clearly communicated in various representations (Kabael & Akin, 2018). Using quantitative reasoning will improve the students’ ability for everyday life, numbers and the mastery of numbers to understand and solve problems. Mental calculations are better, calculations using a tool but not the brain are better, and solutions to the problem can be assessed (Mkhatshwa, 2020). In addition we stated that quantitative thinking is a very important factor in learning mathematics at schools in (Tallman & Frank, 2020; Weber et al., 2014).

Each student's knowledge has characteristics. In the processing of knowledge, human features impair mental function for producing ideas and fail to show if the completion process is right. Via mental exercises, which involve activities in order to define quantities and relate those quantities, strategic methods are then selected and implemented. The attributes of students in thought functions of information processing in response to challenges are called cognitive types (Mefoh et al., 2017). A cognitive style is closely associated with thinking, for reasoning is a cognitive activity. This suggests that the cognitive style will affect the student's quantitative reasoning and bring it to a variety of topics where a topic is generalized. The focus of the research is on field-independent cognitive types and field-dependent styles.

Other cognitive styles developed by Witkin (Son et al., 2020), based on
continuous world analysis, are field-dependent and field-independent cognitive styles. Individuals with a field-independent cognitive styling are more analytical and analyze patterns based on their components in order to respond to stimuli using their own perception. While people with a cognitive style depends on the field tend to react to stimulation using environment as the basis and to see a pattern as a whole, they are not partly divided (Arnup et al., 2013). Several studies have been conducted related to quantitative reasoning (Budiarto & Siswono, 2020; Fu’adiah, 2018; Syarifuddin, 2021). The diversity of research on quantitative reasoning, but no one has studied quantitative reasoning in terms of cognitive style. Consequently, in this research, our goal is to construct a study to show the role of quantitative reasoning as an interesting mathematical thought in the resolution of mathematical problems.

METHOD

In this paper, the role of quantitative reasoning in solving mathematical problems is described in greater depth. Consequently, qualitative research was the approach used to achieve the research objectives. The study consisted of 97 students from South Sulawesi, Indonesia, a public middle school, of whom 28 are male and 69 are female. The participants selected were based on the findings of the GEFT instrument and the Mathematical Ability Test. The participants included in the test. They are classified into field-independent or field-dependent cognitive types and are based on the instrument scores of the Mathematics Ability Test. The criteria used in the selection of subjects, namely subjects with a field-dependent cognitive style (FD) have a score interval of 1-9 out of a total score of 18. While subjects with a field-independent cognitive style (FI) have a score of 10 – 18. The instruments for this research were the group embedded figures test (GEFT), mathematics ability testing (MAT), and mathematical problem-solving testing.

The data analysis process starts from data collection until the researcher completes the task in the field. When the researcher began to collect data, analysis was carried out on the questions asked based on the subject’s responses. To analyze qualitative data, Miles et al. (2018) grouped them into three stages of activity, namely data reduction, data presentation, and drawing conclusions. However, the data analysis process in this study was carried out in steps, namely analyzing the data, checking the credibility of the data by using the time triangulation technique, performing data reduction, data interpretation, and drawing conclusions.

RESULT AND DISCUSSION

1. Description of the Research Participant

The collection of participant was undertaken by 97 grade VIII students of a public middle school. The subject chosen in this analysis referred to cognitive style variations. The cognitive types of students
in this sample were clustered according to the outcomes achieved with the Embedded Group Figure Test (GEFT) instrument from the cognitive style testing results. This represents students' cognitive type data in Table 1.

**Table 1. Students' Cognitive Style**

<table>
<thead>
<tr>
<th>Cognitive Style</th>
<th>Male Students</th>
<th>Female Students</th>
<th>Amount</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field-Independent</td>
<td>17</td>
<td>37</td>
<td>54</td>
<td>55.6%</td>
</tr>
<tr>
<td>Field-Dependent</td>
<td>11</td>
<td>32</td>
<td>43</td>
<td>44.4%</td>
</tr>
<tr>
<td><strong>Amount</strong></td>
<td><strong>28</strong></td>
<td><strong>69</strong></td>
<td><strong>97</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

Furthermore, the findings of a mathematics ability test were also evaluated in order to achieve a prospective participant equal capacity. After giving the Mathematics ability test and cognitive style test to the prospective participant, two students of the same gender were chosen as the participant of this study, namely male gender, with the same level of Mathematics ability, but with different cognitive styles. Table 2 summarizes the characteristics of the two participants with regard to math and cognitive styles.

**Table 2. Characteristic Participant**

<table>
<thead>
<tr>
<th>Initial participants</th>
<th>MAT Scorer</th>
<th>GEFT Scorer</th>
<th>Cognitive Style Category</th>
<th>Participant Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRA</td>
<td>80</td>
<td>12</td>
<td>Field-Independent</td>
<td>SLFI</td>
</tr>
<tr>
<td>FAR</td>
<td>80</td>
<td>8</td>
<td>Field-Dependent</td>
<td>SLFD</td>
</tr>
</tbody>
</table>

2. **Description of Students' Field-Independent (SLFI) Quantitative Reasoning in Solving Mathematical Problems**

In the following, a quantitative reasoning worksheet for the SLFI subject for solving mathematical problems is presented in Figure 1.

![Figure 1. Result of Problem Solving by SLFI](image)

SLFI understands quantitative information by conveying all the information contained in the problem situation, starting with reading the problem, then presenting the reading. From the reading results, it is then stated the elements that are known and what is being asked
using random language that is simple and easy to understand. From the information presented, reasons are then given by using the words in the problem and linking the facts in the question between what is known and what is being asked. This SLFI understanding activity is in line with what was stated by (Herbst & Chazan, 2020) that understanding is included in the exemplary category, namely finding certain examples or illustrations of a concept or principle. Furthermore, SLFI makes examples of problems that are similar to the problems faced by utilizing previous knowledge and then states that there are similarities between the examples made and the problems at hand. This is in accordance with the opinion of (Ramful & Ho, 2015) that quantitative reasoning is a mental activity related to analyzing the relationship between quantity and situations and creating new situations.

SLFI uses its quantitative reasoning by making a relationship between the magnitude of each problem given by providing an explanation of the relationship. On the other hand, SLFI also justifies its reasoning by expressing arguments precisely and logically, using language that is simple and easy to understand. For cases other than a conditional relationship, it does not create a relationship, but SLFI does see a number pattern. This is in accordance with the opinion of (Ackerman & Thompson, 2017; Ramful & Ho, 2015) who state that individuals who work with these quantities and quantity relationships, then these individuals involve their quantitative reasoning. SLFI looks for the same relationship between two or more problems. It finds a constant relationship. Then, from the relationship obtained, SLFI gave verbal reasons. This SLFI activity is in line with the opinion of Weber et al. (2014) that generalization activities are mental actions that end in verbal activities. Furthermore, SLFI establishes a procedure for detecting the relationship between quantities using a different method for each given problem. On contextual problems, SLFI explains the procedure for solving them, starting with rewriting the information contained in the questions, then building its understanding of the relationship, namely the relationship between quantities. Then SLFI provides reasons related to the relationship that is built to solve the problem at hand. The procedural activities carried out by SLFI is opinion by Li et al., (2020); Mkhatshwa, (2020) state that quantitative reasoning plays a key role in the problem-solving process and this skill ensures that problem-solving is carried out efficiently.

SLFI generates generic forms using different means for each given problem. First, the problems given are related to contextual problems. To get the general form, field independent male subjects focus on the relationship between quantities by involving comparative relationships to get patterns and verbally expressing the resulting patterns. He then manipulated the obtained patterns to produce a
generalized form, the resulting general form being written in algebraic form. Second, the problem of patterns in the form of images. SLFI produces the general form through the formula Un = a + (n-1)b. SLFI's activities are based on the opinion by Mouhayar (2018) that students use patterns they have previously identified to generate generalizations. Where generalizations can be made verbally or symbolically. For symbolic, that is by building algebraic relationships between the patterns it gets.

3. Description of Students' Field-Dependent (SLFD) Quantitative Reasoning in solving mathematical problems

In the following, a quantitative reasoning worksheet for the SLFD subject for solving mathematical problems is presented in Figure 2.

![Figure 2. Result of Problem Solving by SLFD](image-url)

SLFD started the activity by reading the questions and presenting the reading results. The reading results then convey all the information orally and coherently, according to the order of information about the questions about what is known and what is being asked. Then give reasons by using the words in the problem and linking the facts in the problem between what is known and what is asked. SLFD relates the magnitude of each question given by providing an explanation of the relationship in question. On the other hand, he justifies his reasoning by expressing arguments in a precise and logical manner, using language that is simple and easy to understand. The activities carried out by SLFD are in line with the results of research by Kuo et al. (2013); Ramful & Ho (2015) explained that students’ quantitative reasoning in the problem-solving process begins with reading the problem.

SLFD looks for the same relationship between two or more quantities of a given problem. SLFD finds a constant relationship. Then, from the relationship he got, he gave logical reasons verbally. This is in accordance with the opinion of (Ramful & Ho, 2015; Weber et al., 2014) which states that individuals who work with these quantities and quantity relationships, then these individuals involve...
their quantitative reasoning. The SLFD settlement procedure begins by identifying the quantity and by restating the information related to the problematic quantity. Furthermore, SLFD uses quantitative reasoning by connecting quantities through the concept of value comparisons. This concept is used to solve the problem at hand. The procedural activities carried out by SLFD are in line with the opinion of Psycharisis & Kallia (2017) that quantitative reasoning plays a key role in the problem-solving process and these skills ensure that problem solving is carried out efficiently.

SLFD states that there are patterns that exist in certain problems, expressing them verbally. Then he gave a proper and logical reason. It also tests the detected pattern where the test results show that the resulting pattern will always last. This SLFD activity is in line with the opinion of Ellis et al., 2019 that generalization activities are mental actions carried out through verbal activities. SLFD also generates general forms in a different way for each given problem. First, the problems given are related to contextual problems. SLFD takes a general form, from a pattern used to solve a particular problem, by using the concept of an opinion. This activity is carried out repeatedly in the process of solving a specific problem of a given problem. Furthermore, SLFD is unable to apply to new situations by making questions that are similar to the problems.

4. Similarities and Differences of Students' Field-Independent (SLFI) and Field-Dependent (SLFD) Quantitative Reasoning in Solving Mathematical Problems

In detail the forms and differences in SLFI and SLFD quantitative reasoning in solving mathematical problems are described in Table 3 below.

<table>
<thead>
<tr>
<th>Similarity</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presenting the elements that are known and questioned in full accordance with the information in accordance with the assigned task. Related the problem situation to the problem obtained previously by stating that the problems faced were related to the comparison of material values.</td>
<td>SLFD understands problems by putting things that are known and asked in their own language and are easy to run randomly, while SLFD understands problems by putting things that are known and what is asked in the context of the problem (coherently). SLFI can make sample questions based on the questions given and state the elements of their similarity, while SLFD cannot make example questions by reason of forgetting.</td>
</tr>
</tbody>
</table>
Quantitative reasoning is reasoning that uses basic mathematical skills in analyzing and interpreting quantitative information about a problem in everyday life (Elrod, 2014; Ramful & Ho, 2015). From this statement, it can be concluded that the quantitative reasoning process uses basic mathematical skills and analyzes quantitative information to solve contextual problems. It can be concluded that quantitative reasoning is reasoning that uses mathematical abilities to obtain quantitative information. The role of quantitative reasoning is directly useful in everyday life and applies mathematical skills to interpreting data and solving problems in real-world contexts.

CONCLUSION

The results from this analysis show that in the process of solving mathematical problems, the quantitative argumentation of field-independent and field-dependent subjects focuses on quantity. Subject Field-Independent may expose issues or circumstances close to the given problem and use quantitative reasoning such as the relation between material amount and quantity and the relation between material quantity. Subjects use quantitative thinking in order to seek fixed links, namely the matrix, to select the ratio of the amount and then to calculate quantities and the ratio of the quantities to the solution and to the result.

The subject Field-Dependant is able to understand the problem context in solving mathematical problems through quantitative thinking, identify quantities within the problem, focus on quantities within the problem context, determine relationships between quantities, analyze
quantity changes with regard to the problem, draw up plans for quantitative relation(s).

The role of quantitative reasoning will help students concentrate on the quantity and quantity of information in general. The results are important to teachers as an input material and the promotion of mathematical education. The quantitative reasoning provided by students is developed and utilized in this situation in the learning process. Other researchers should also be investigating other issues relating to middle school students' quantitative reasoning.

REFERENCES


NCTM.


