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Edge Monophonic Metric Dimension of Some Corona Graphs

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ABSTRACT

This study introduces the concept of the edge monophonic metric dimension of a graph, which is the minimum cardinality among all such edge monophonic resolving sets. The research aims to define this parameter using vertex-to-edge monophonic distance and determine its values for path graph P_n, cycle graph C_n, and corona product graphs, including $P_n \odot K_1$, $C_n \odot K_1$, $K_1 \odot P_n$, and $K_1 \odot C_n$. A theoretical and deductive method is employed by formulating definitions, establishing properties, and proving the main results. The exact values of the edge monophonic metric dimension are obtained, that is $em\dim(P_n) = 1$, $em\dim(C_n) = 2$, and several corona product graphs, including $em\dim(P_n \odot K_1)$, $em\dim(C_n \odot K_1)$, $em\dim(K_1 \odot P_n)$, and $em\dim(K_1 \odot C_n)$. The results provide new insights into distance-based graph parameters and contribute to the development of graph theory.

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1. Introduction

Graph theory is an important branch of discrete mathematics that is widely used to model various real-world problems in computer science, communication networks, biology, and even social sciences. A graph is generally defined as a structure consisting of a set of vertices and a set of edges, where the edges connect pairs of vertices (maybe empty). This concept was first introduced by Leonhard Euler in 1736 through the famous Königsberg Bridge problem (Chartrand et al., 2024).

The corona product of two graphs G and H, denoted by $G \odot H$, is defined as the graph obtained by taking one copy of G and n copies of H, where $n = |V(G)|$,

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and then joining each vertex v_i of $V(G)$ to every vertex in the corresponding copy H_i of H, for $1 \leq i \leq n$. In other words, for each vertex $v_i \in V(G)$, $1 \leq i \leq n$, a copy H_i of H is added, and v_i is adjacent to all vertices in H_i (Frucht & Harary, 1970).

One of the rapidly developing topics in graph theory is the metric dimension. The metric dimension of a graph was first introduced by Slater in 1975 and further developed by Harary and Melter in 1976. This concept is related to the solution set, which is the set of vertices that can distinguish all vertices in the graph based on their shortest path distances. From this concept the idea of metric dimension, which is understood as the minimum size of the solution set in a graph. In other words, the metric dimension represents the smallest number of vertices required for each vertex in the graph to be uniquely distinguished by its shortest path distance to the vertices in the set (Harary & Melter, 1976; Slater, 1975).

As studies on metric dimension continued to develop, in 2018 Kelenc and his colleagues introduced the concept of edge metric dimension. While the previously established metric dimension focuses on distinguishing the vertices of a graph based on their distances to a given resolving set, the edge metric dimension concentrates on distinguishing the edges of a graph. In this concept, the distance between a vertex and an edge is defined as the minimum distance between the vertex and the endpoints of the edge. Formally, let $G = (V, E)$ be a connected graph. For a vertex $v \in V(G)$ and an edge $e = uw \in E(G)$, the distance between v and e is defined as $d(v, e) = min\{d(v, u), d(v, w)\}$. Let $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$. The representation of an edge $e \in E(G)$ with respect to W is given by $r(e \mid W) = (d(w_1, e), d(w_2, e), \dots, d(w_k, e))$. The set W is called an edge resolving set if for every pair of distinct edges $e, f \in E(G)$ with $e \neq f$, then $r(e \mid W) \neq r(f \mid W)$. The edge metric dimension of G, denoted by $edim(G)$, is the minimum cardinality of such a set W (Albab & Zuhri, 2023; Ilmayasinta, 2019; Kelenc et al., 2016; Wei et al., 2020; Zhu et al., 2021). Research on the metric dimension and the edge metric dimension of graphs has attracted significant attention, particularly in the study of graph operations such as the corona product (Peterin & Yero, 2020; Rinurwati et al., 2021, 2016, 2017a, 2017b; Rinurwati & Nabila, 2021; Rinurwati, Setyawati, et al., 2024; Rinurwati, Sundusia, et al., 2024; Rosyidah & Rinurwati, 2021).

Studies on distances in graphs have also undergone significant development. One such concept is the monophonic distance, defined as the distance derived from the longest monophonic path, where a monophonic path is a path that contains no chords. For any two vertices $u, v \in V(G)$, the monophonic distance between u and v, denoted by $d_m(u, v)$, is defined as the length of the longest monophonic path connecting u and v (Santhakumaran & Titus, 2012). This concept of monophonic distance was later extended to measure the distance from a vertex to an edge in a graph (Asir & Athisayanathan, 2016).

In 2023, Priya, Muthu, and Beula introduced and developed the concept of monophonic metric dimension based on monophonic distance. The monophonic metric dimension is defined as the minimum size of a set of vertices that can distinguish every other vertex in the graph based on their monophonic distances. Such a set of vertices is called a monophonic resolving set. Formally, let $W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)$. For any vertex $v \in V(G)$, the monophonic representation of v with respect to W is defined as $r_m(v \mid W) = (d_m(v, w_1), d_m(v, w_2), \dots, d_m(v, w_k))$. The set W is called a monophonic resolving set if for every pair of distinct vertices $u, v \in V(G)$ with $u \neq v$, then $r_m(u \mid W) \neq r_m(v \mid W)$. The monophonic metric dimension of G, denoted by $\dim_m(G)$, is the minimum cardinality of such a set W

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(Lathifah & Rinurwati, 2025; Priya & Beula, 2023).

From the existing literature, it can be observed that research on monophonic metric dimension has thus far focused on using monophonic distances between pairs of vertices. Meanwhile, studies on monophonic metric dimensions using monophonic distance from vertices to edges have not yet been found. This indicates a relevant research gap, particularly in combining the concept of monophonic vertex–edge distance in graphs with the theory of edge metric dimension.

The main contribution of this research is the introduction and development of the concept of edge monophonic metric dimensions, which extends the monophonic metric dimension from distinguishing vertices to distinguishing edges based on monophonic vertex to edge distances. This study provides the formal definition of the edge monophonic metric dimension and investigates its properties for several fundamental classes of graphs, including path and cycle graphs. Furthermore, the concept is extended to corona product graphs, where the edge monophonic metric dimension is determined for several classes of corona graphs. These results contribute to the development of metric dimension theory in graph theory.

Based on this, the present research will focus on the edge monophonic metric dimension of path graphs P_n and cycle graphs C_n, and will further extend the investigation to the edge monophonic metric dimension of corona product graphs that is $P_n \odot K_1$, $C_n \odot K_1$, $K_1 \odot P_n$, and $K_1 \odot P_n$. This research is expected to broaden the understanding of metric dimensions in graphs and open opportunities for further development of graph theory in a wider context.

2. Method

This study employs a theoretical and deductive approach to determine the edge monophonic metric dimension of several graph classes. The study begins by defining the monophonic distance, where $d_m(u, v)$ is the length of the longest monophonic path between two vertices u and v, with a monophonic path containing no chord (Santhakumaran & Titus, 2012). Based on this concept, the monophonic vertex-edge distance, edge monophonic representation, and edge monophonic resolving set are formally defined.

Fundamental properties of edge monophonic resolving sets are then established to derive general lower and upper bounds for the edge monophonic metric dimension. Exact values are subsequently determined for path graphs P_n and cycle graphs C_n by constructing edge monophonic representations and identifying minimum resolving sets, with the results presented as theorems and rigorous proofs. The analysis is further extended to the corona product graphs $P_n \odot K_1$, $C_n \odot K_1$, $K_1 \odot P_n$, and $K_1 \odot P_n$. Finally, graphs attaining specific values or bounds are characterized based on their structural properties.

3. Results and Discussion

In this subsection, we present several fundamental definitions that are essential for the development of our results and form the basis for the analysis of the edge monophonic metric dimension. The concept of monophonic distance plays a central role in this study.

This notion differs from the classical shortest-path distance and captures structural properties of graphs related to induced paths. Based on this concept, the

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monophonic distance can be naturally extended to measure the distance between a vertex and an edge. Specifically, for any vertex $v \in V(G)$ and any edge $e = xy \in E(G)$, where $x, y \in V(G)$, the monophonic distance from v to e is defined as $d_m(v, e) = min\{d_m(v, x), d_m(v, y)\}$, which reflects the closest interaction between the vertex and the endpoints of the edge under monophonic distance, called vertex-edge monophonic distance.

Furthermore, let $S = \{s_1, s_2, \dots, s_k\} \subseteq V(G)$, where $1 \leq k \leq |V(G)|$. The edge monophonic representation of an edge $e \in E(G)$ with respect to the set S, denoted by $r_{em}(e \mid S)$, is defined as the ordered k-tuple $r_{em}(e \mid S) = (d_m(s_1, e), d_m(s_2, e), \dots, d_m(s_k, e))$. This representation encodes the relative position of the edge e with respect to the vertices in S under the monophonic distance. A set $S \subseteq V(G)$ is called an edge monophonic resolving set if every pair of distinct edges $e_i, e_j \in E(G)$ has distinct edge monophonic representations, that is, $r_{em}(e_i \mid S) \neq r_{em}(e_j \mid S)$. In other words, the set S can uniquely distinguish all edges in the graph based on their monophonic distances to the vertices in S. Finally, the minimum cardinality among all such edge monophonic resolving sets is called the edge monophonic metric dimension of G, and is denoted by $em\dim(G) = \min\{|S| \mid S \subseteq V(G), S \textit{ is an edge monophonic resolving set}\}$.

To establish the main result, we first present the following lemma which will be useful in the subsequent proofs,

Lemma 1. If $G = (V, E)$ be a connected graph and let $S \subseteq V(G)$. If $e = xy$ and $f = uv$ are two distinct edges in $E(G)$ such that $x, y, u, v \in S$, then $r_{em}(e \mid S) \neq r_{em}(f \mid S)$.

Proof. By Definition 1, $d_m(v, e) = \min\{d_m(v, x), d_m(v, y)\}$. Since $x, y \in S$, we obtain $d_m(x, e) = 0$ and $d_m(y, e) = 0$. Thus, by definition of edge monophonic representation, the edge monophonic representation $r_{em}(e \mid S)$ contains zeros precisely in the coordinates corresponding to x and y. Similarly, since $u, v \in S$, $r_{em}(f \mid S)$ contains zeros precisely in the coordinates corresponding to u and v. Because $e \neq f$, we have $\{x, y\} \neq \{u, v\}$. Hence the zero positions in the two representation vectors are different, implying $r_{em}(e \mid S) \neq r_{em}(f \mid S)$. ■

In **Lemma 1**, it guarantees that edges induced by vertices in S have distinct representations. This observation leads directly to the following lemma.

Lemma 2. If $G = (V, E)$ be a connected simple graph. Then $V(G)$ is an edge monophonic resolving set of G.

Proof. Let $e = xy$ and $f = uv$ be two distinct edges in $E(G)$. Since G is simple, we have $\{x, y\} \neq \{u, v\}$. By definition of vertex-edge monophonic distance, $d_m(v, e) = \min\{d_m(v, x), d_m(v, y)\}$. In particular, $d_m(x, e) = 0$ and $d_m(y, e) = 0$. Thus, in the representation $r_{em}(e \mid V(G))$, the coordinates corresponding to x and y are zero. Similarly, in $r_{em}(f \mid V(G))$, the coordinates corresponding to u and v are zero. Since $\{x, y\} \neq \{u, v\}$, the positions of zeros in the two representation vectors differ. Hence, $r_{em}(e \mid V(G)) \neq r_{em}(f \mid V(G))$. Therefore, $V(G)$ is an edge monophonic resolving set of G. ■

To clarify the concept of edge monophonic representation and resolving sets, we present the following illustrative example.

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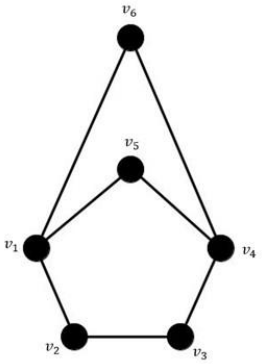

Figure 1. Graph G

Example 1. Let G be the graph as shown in **Figure 1**. Based on the definition of the **edge monophonic metric dimension**, it is obtained that $em\dim(G) = 3$. One of the edge monophonic resolving sets of graph G is $S = \{v_2, v_3, v_5\}$. The edge monophonic representation of each edge in graph G with respect to the set S is determined based on the monophonic distance from each vertex in S to the corresponding edge. Thus, the edge monophonic representations are obtained as follows:

$$r_{em}(v_1v_2 \mid S) = (0,1,1), \quad r_{em}(v_2v_3 \mid S) = (0,0,3),$$
$$r_{em}(v_3v_4 \mid S) = (1,0,1), \quad r_{em}(v_4v_5 \mid S) = (3,1,0),$$
$$r_{em}(v_5v_1 \mid S) = (1,3,0), \quad r_{em}(v_1v_6 \mid S) = (1,3,1),$$
$$r_{em}(v_4v_6 \mid S) = (3,1,1).$$

Each edge in graph G has a distinct edge monophonic representation with respect to S. Therefore, the set $S = \{v_2, v_3, v_5\}$ is an edge monophonic resolving set of graph G. Since there is no edge monophonic resolving set with cardinality less than three, it follows that $em\dim(G) = 3$.

We begin with the simplest connected graph structure, namely the path graph. The following theorem determines the exact value of the edge monophonic metric dimension of P_n.

Theorem 1. For path graph P_n with vertex set $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(P_n) = e_{ij} = \{v_i\, v_j \mid 1 \le i < j = i+1 \le n-1\}$. Then for $n \ge 3$ the edge monophonic metric dimension is

$$em\dim(P_n) = 1$$

Proof. Consider the set $S = \{v_1\}$. Since P_n is a path, between any two vertices there is a unique path, which contains no chord. Hence, the monophonic distance between any two vertices of P_n is equal to their standard distance. For each edge $e_i = v_iv_{i+1}$ $(1 \le i \le n-1)$, we have $d_m(v_1, e_i) = \min\{d_m(v_1, v_i), d_m(v_1, v_{i+1})\}$. Thus, the representations of the edges $e_1, e_2, \dots, e_{n-1}$ with respect to are $r_{em}(e_i \mid S) = i$, $1 \le i \le n-1$

These values are pairwise distinct. Therefore, S distinguishes every pair of distinct edges of P_n. Consequently, is an edge monophonic resolving set, and hence $em\dim(P_n) \le 1$.

On the other hand, the empty set cannot be an edge monophonic resolving set. Indeed, for any two distinct edges $e_i, e_j \in E(P_n)$, their edge monophonic representations with respect to the empty set are both the empty tuple, that is,

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$r_{em}(e_i \mid \emptyset) = r_{em}(e_i \mid \emptyset) = ()$. Hence, the empty set cannot distinguish any pair of distinct edges. Therefore, $em\dim(P_n) \geq 1$.
Then, we can conclude

$$em\dim(P_n) = 1.$$ ■

The path graph provides the simplest linear structure. We now extend our analysis to cycle graphs, where the presence of a closed structure introduces different distinguishing patterns.

Theorem 2. For the cycle graph C_n with $n \geq 3$, the edge monophonic metric dimension is

$$em\dim(C_n) = 2$$

Proof. Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{\, e_i = v_i v_{i+1} \mid 1 \leq i \leq n-1 \,\} \cup \{e_n = v_n v_1\}$.Consider the set $S = \{v_1, v_2\}$. In the cycle graph C_n, between any two distinct vertices there are exactly two internally disjoint paths. Since a monophonic path must contain no chord, the longest monophonic path between two vertices is obtained by taking the longer of the two paths around the cycle. Hence, for any vertex $v_i, v_k \in V(C_n)$,

$$d_m(v_i, v_k) = \begin{cases} 1, & \text{if } v_i, v_k \; are \; adjencent, \\ \max\{|k-i|, n-|k-i|\}, & \text{if } v_i, v_k \; are \; non-adjencent, \end{cases}$$

and let $e_i = v_i v_{i+1}$ be an arbitrary edge of C_n. By Definition 1, $d_m(v_j, e_i) = \min\{d_m(v_j, v_i), d_m(v_j, v_{i+1})\}, j = 1,2$. Thus, the edge monophonic representation of e_i with respect to S is $r_{em}(e_i \mid S) = (d_m(v_1, e_i), d_m(v_2, e_i))$. Fix a vertex $v \in V(C_n)$. The values $d_m(v, e_i)$ depend only on the cyclic position of e_i relative to v. Along the cycle, these values increase stepwise from the two edges incident with v until reaching a maximum at the opposite side of the cycle, and then decrease symmetrically. Thus, for a fixed vertex v, two distinct edges can have equal $d_m(v, e_i)$ only if they are symmetric with respect to v. Now suppose $e_i \neq e_j$.

- If $d_m(v_1, e_i) \neq d_m(v_1, e_j)$, then their representations differ.
- If $d_m(v_1, e_i) = d_m(v_1, e_j)$, then e_i and e_j are symmetric with respect to v_1. Since v_2 is adjacent to v_1, this symmetry does not hold with respect to v_2, and therefore $d_m(v_2, e_i) \neq d_m(v_2, e_j)$.

Hence for all $i \neq j$, $r_{em}(e_i \mid S) \neq r_{em}(e_j \mid S)$. Therefore $S = \{v_1, v_2\}$ is an edge monophonic resolving set, and $em\dim(C_n) \leq 2$. Assume that there exists an edge monophonic resolving set S with $|S| = 1$, say $S = \{v\}$. Since C_n is vertex-transitive and symmetric, there exist two distinct edges $e_i \neq e_j$ that are symmetrically located with respect to v. Because monophonic distance depends only on the length of the longer induced path, these two edges satisfy $d_m(v, e_i) = d_m(v, e_j)$. Consequently, $r_{em}(e_i \mid S) = r_{em}(e_j \mid S)$, which contradicts the assumption that S is an edge monophonic resolving set. Hence, no edge monophonic resolving set of cardinalities one exists, and therefore $em\dim(C_n) \geq 2$. Therefore, we can conclude that $em\dim(C_n) = 2$. ■

The results obtained for path and cycle graphs provide a fundamental understanding of the edge monophonic metric dimension on basic graph structures. We now generalize this study to graph operations, particularly the corona product. In particular, the case K_1 is considered as one of the component

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graphs of the corona product. Since K_1 consists of a single vertex with no edge, the corona product with K_1 attaches one pendant vertex to every vertex of the original graph. This structure allows the effect of the corona operation on the edge monophonic metric dimension to be analyzed explicitly. The following theorem determines the edge monophonic metric dimension of the corona graph $P_n \odot K_1$.

Theorem 3. For corona graph $P_n \odot K_1$ with $n \geq 3$, then the edge monophonic metric dimension is

$$em\dim(P_n \odot K_1) = 2$$

Proof. Let $G = P_n$ with $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{e_{i,i+1} = v_i\, v_{i+1} \mid 1 \leq i \leq n-1\}$. Let $H = K_1$ with $V(H) = \{u_1\}$. Since K_1 has no edges, we have $E(H) = \emptyset$. Because $| V(G) |= n$ and $| V(H) |= 1$, the vertex set of the corona graph is $V(P_n \odot K_1) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{i1} \mid 1 \leq i \leq n\}$. The edge set is divided into $E_1 = \{e_{i,i+1} = v_i\, v_{i+1} \mid 1 \leq i \leq n-1\}$, $E_2 = \emptyset$ (since K_1 has no edges), and $E_3 = \{g_{i1} = v_i\, u_{i1} \mid 1 \leq i \leq n\}$. Thus, $E(P_n \odot K_1) = E_1 \cup E_3$. Consider the set $S = \{u_{11}, u_{n1}\}$. We prove that S distinguishes all edges.

Case (1) Edges in E_3. Take an edge $g_{i1} = v_i\, u_{i1}$, because the graph is a path with a leaf attached to each v_i, we obtain $d_m(u_{11}, g_{i1}) = i$ and $d_m(u_{n1}, g_{i1}) = n - i + 1$. Hence, $r_{em}(g_{i1} \mid S) = (i, n - i + 1)$.

Case (2) Edges in E_1. Take an edge $e_{i,i+1} = v_i\, v_{i+1}$. Then, $d_m(u_{11}, e_{i,i+1}) = i$ and $d_m(u_{n1}, e_{i,i+1}) = n - i$. Thus, $r_{em}(e_{i,i+1} \mid S) = (i, n-i)$. For the same index i, $(i, n - i + 1) \neq (i, n-i)$, so edges in E_3 and E_1 are distinguished. If $i \neq j$, then the first coordinate differs.

Therefore, every edge in $E_1 \cup E_3$ has a distinct representation with respect to S. Hence, $em\dim(P_n \odot K_1) \leq 2$. Assume that only one vertex is chosen. Since the graph is symmetric (for $n \geq 3$), there exist two different edges having equal monophonic distance to that single vertex (for example, two edges symmetrically located with respect to the chosen vertex). Thus, one vertex is not sufficient to distinguish all edges. Therefore, $em\dim(P_n \odot K_1) \geq 2$.

We conclude that $em\dim(P_n \odot K_1) = 2$. ■

After determining the edge monophonic metric dimension of the corona graph $P_n \odot K_1$, we now consider the corona product $C_n \odot K_1$.

Theorem 4. For corona graph $C_n \odot K_1$, with $n \geq 3$, then the edge monophonic metric dimension is

$$em\dim(C_n \odot K_1) = \begin{cases} 2 & , n = 3 \text{ and } n \geq 4 \text{ even} \\ 3 & , n \geq 4 \text{ odd} \end{cases}$$

Proof. Let $G = C_n$ be a cycle graph with $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{v_i v_{i+1} \mid 1 \leq i \leq n-1\} \cup \{v_n v_1\}$. Let $H = K_1$ with vertex set $\{u_1\}$. Then, $V(C_n \odot K_1) = \{v_i \mid 1 \leq i \leq n\} \cup \{u_{i1} \mid 1 \leq i \leq n\}$. The edge set is E_1: cycle edges $v_i v_{i+1}$ and $v_n v_1$, $E_2 = \emptyset$, and $E_3 = \{g_{i1} = v_i\, u_{i1} \mid 1 \leq i \leq n\}$. Thus $C_n \odot K_1$ is a unicyclic graph (one cycle with n pendant vertices). Since $C_n \odot K_1$ is a unicyclic graph, its unique cycle is C_n. The cycle C_n contains no chord, since there is no edge joining any two nonconsecutive vertices of the cycle. Moreover, each pendant vertex has degree one and therefore cannot create a chord. Hence, the relevant paths in $C_n \odot K_1$ are chordless, and the monophonic distance between two vertices is determined by the length of the longest chordless path connecting them.

Case 1: $n = 3$. The graph is fully symmetric. Take $S = \{u_{11}, u_{21}\}$. By direct

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computation, all edges in $E_1 \cup E_3$ have distinct edge monophonic representations with respect to S. Thus, $em\dim(C_3 \odot K_1) = 2$.

Case 2: $n \geq 4$ $even$. Choose two pendant vertices located at opposite positions on the cycle $S = \{u_{11}, u_{\frac{n}{2}+1,1}\}$. Since n is even, these two vertices are antipodal. For every pendant edge $g_{i1} \in E_3$, then $r_{em}(g_{i1} \mid S) = \left(d_m(u_{11}, g_{i1}), d_m\left(u_{\frac{n}{2}+1,1}, g_{i1}\right)\right)$. Because of the perfect symmetry of an even cycle, distances measured from two antipodal points uniquely determine the position of each vertex on the cycle. Hence all pendant edges obtain distinct representations. Similarly, for every cycle edge $e_{i,i+1} \in E_1$, the pair of distances to the two antipodal vertices is also unique. Therefore, all edges have distinct representations with respect to S, and $em\dim(C_n \odot K_1) \leq 2$. Since a single vertex cannot distinguish symmetric edges, we conclude $em\dim(C_n \odot K_1) = 2$.

Case 3: $n \geq 4$ odd. Assume we choose any two vertices, for example $S = \{u_{11}, u_{k1}\}$. Because n is odd, there is no exact antipodal vertex. The cycle possesses a reflective symmetry that leaves two edges symmetric with respect to the chosen pair. Consequently, there always exist two distinct edges having identical distance pairs to any two chosen vertices. Thus, $em\dim(C_n \odot K_1) \geq 3$. Now choose three pendant vertices, for example $S = \{u_{11}, u_{21}, u_{31}\}$. With three reference points on an odd cycle, all symmetries are broken. No two distinct edges can share the same triple of distances. Hence, $em\dim(C_n \odot K_1) \leq 3$. Therefore, $em\dim(C_n \odot K_1) = 3$.
Then we can conclude that

$$em\dim(C_n \odot K_1) = \begin{cases} 2 & ,n = 3 \text{ and } n \geq 4 \text{ even} \\ 3 & ,n \geq 4 \text{ odd}. \end{cases}$$

■

After establishing the result for $C_n \odot K_1$, we proceed to investigate the edge monophonic metric dimension of the corona graph $K_1 \odot P_n$.

Theorem 5. For corona graph $K_1 \odot P_n$, with $n \geq 3$, then the edge monophonic metric dimension is

$$em\dim(K_1 \odot P_n) = \begin{cases} n & ,n = 3 \\ n-1 & ,n \geq 4 \end{cases}$$

Proof. Let $V(K_1) = \{v\}$ and $V(P_n) = \{u_1, u_2, \dots, u_n\}$, with edges $u_i u_{i+1}$, $1 \leq i \leq n-1$. Let $G = K_1 \odot P_n$, then $V(G) = \{v, u_1, \dots, u_n\}$ and $E(G) = \{u_i u_{i+1} \mid 1 \leq i \leq n-1\} \cup \{vu_i \mid 1 \leq i \leq n\}$, with n pendant edges $e_i = vu_i$, and $n-1$ path edges $f_i = u_i u_{i+1}$. Since G is a tree, it is connected and contains no cycle. Therefore, there exists a unique path between every pair of vertices in G. This unique path contains no chords and hence is a monophonic path. Consequently, the longest monophonic path between any two vertices is the unique path connecting them, so the monophonic distance coincides with the usual distance. Thus, $d_m(s, e) = min\{d(s, x), d(s, y)\}$ for $e = xy$.
Case 1: $n = 3$. Assume $| S | \leq 2$. Then at least one vertex among u_1, u_2, u_3 is not in S, that u_i. Consider edges $e = vu_i$, $f = u_{i-1}\, u_i$ or $u_i\, u_{i+1}$. For every $s \in S$,

$$d_m(s, e) = d(s, u_i)$$
$$d_m(s, f) = min\{d(s, u_i), d(s, u_{i\pm1})\}.$$

Because S contains at most two vertices, one verifies directly that there exists a pair

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of edges with identical representations. Hence no set of size 2 resolves all edges, so $em\dim(G) \geq 3$. Take $S = \{u_1, u_2, u_3\}$, each pendant edge vu_i is uniquely identified since $d_m(u_i, vu_i) = 0$ while for $j \neq i$, $d_m(u_i, vu_j) = d(u_i, u_j) \geq 1$. Path edges are also distinguished since each has a unique endpoint in S at distance 0. Thus S resolves all edges and $em\dim(G) \leq 3$. Therefore, $emdim(K_1 \odot P_3) = 3 = n$.
Case 2: $n \geq 4$. Let $S = \{u_1, u_2, \dots, u_{n-1}\}$. We show S resolves all edges.

(i) Edge pendant and pendant. For $e_i = vu_i$ and $e_j = vu_j$, $i \neq j$: Since $u_i \in S$, $d_m(u_i, e_i) = 0$ and $d_m(u_i, e_j) = d(u_i, u_j) \geq 1$. Hence distinct.

(ii) Edge path and path. For $f_i = u_i u_{i+1}$ and $f_j = u_j\, u_{j+1}$, $i \neq j$: $d_m(u_i, f_i) = 0$ and $d_m(u_i, f_j) \geq 1$. Thus distinct

(iii) Edge pendant and path. Compare $e_i = vu_i$ and $f_j = u_j u_{j+1}$. If $i \neq j, j+1$, then using $s = u_i$: $d_m(u_i, e_i) = 0$ and $d_m(u_i, f_j) \geq 1$. If $i = j$, then $d_m(u_i, e_i) = 0$ and $d_m(u_i, f_i) = 0$, but since $n \geq 4$, there exists u_{i-1} or u_{i+1} in S giving different distances.

Hence all edges are distinguished and $em\dim(G) \leq n - 1$. Assume $|S| \leq n - 2$. Then at least two vertices among $u_1, \dots, u_n$, say u_i, u_j, are not in S. Consider pendant edges $e_i = vu_i$ and $e_j = vu_j$. For every $s \in S$, $d_m(s, e_i) = d(s, u_i), d_m(s, e_j) = d(s, u_j)$. Since distances in a tree depend only on path lengths and the center v creates symmetric branches, we may choose i, j symmetrically positioned so that $d(s, u_i) = d(s, u_j)$ for all $s \in S$. Thus $r_{em}(e_i \mid S) = r_{em}(e_j \mid S)$, contradiction. Therefore, $|S| \geq n - 1$. For $n \geq 4$, $emdim(K_1 \odot P_n) = n - 1$.
We concluded that,

$$em\dim(K_1 \odot P_n) = \begin{cases} n & ,n = 3 \\ n-1 & ,n \geq 4 \end{cases}$$

■

Having established the edge monophonic metric dimension of $K_1 \odot P_n$, we now extend the analysis to the corona graph $K_1 \odot C_n$.

Theorem 6. For corona graph $K_1 \odot C_n$, with $n \geq 3$, then the edge monophonic metric dimension is

$$em\dim(K_1 \odot C_n) = \begin{cases} n & ,n \in \{3,4\} \\ n-1 & ,n \geq 5 \end{cases}$$

Proof. Let $V(K_1) = \{v\}$ and $V(C_n) = \{u_1, u_2, \dots, u_n\}$ with cycle edges $u_i\, u_{i+1}, 1 \leq i \leq n-1$ and $u_n\, u_1$. Let $G = K_1 \odot C_n$, then $V(G) = \{v, u_1, \dots, u_n\}$ and $E(G) = \{u_i\, u_{i+1}, u_n\, u_1\} \cup \{vu_i \mid 1 \leq i \leq n\}$. Edges consist of n cycle edges f_i, n pendant edges $e_i = vu_i$. Thus total $2n$ edges.
Case 1: $n = 3$. Here $C_3 = K_3$. Thus $G = K_1 \odot C_3$ has 3 pendant edges and 3 cycle edges, total 6 edges. Because of complete symmetry of the triangle, omitting any vertex u_i causes two pendant edges to have identical representations. Hence any resolving set must contain all three u_i. Thus, $em\dim(G) = 3 = n$.
Case 2: $n = 4$. Graph: one central vertex attached to a 4-cycle. The cycle C_4 has strong opposite-vertex symmetry $u_1 \leftrightarrow u_3$ and $u_2 \leftrightarrow u_4$. Assume $|S| \leq 3$, then at least one cycle vertex is not in S. Suppose $u_i \notin S$. Because of 4-cycle symmetry, there exists an opposite vertex u_j whose distance relations to all vertices of S coincide with those of u_i. Hence, the pendant edges e_i and e_j have identical edge representations, which contradicts the assumption that S is an edge monophonic resolving set. Thus $em\dim(G) \geq 4$. Take $S = \{u_1, u_2, u_3, u_4\}$. Each pendant edge vu_i is uniquely determined by $d_m(u_i, vu_i) = 0$. Cycle edges are also distinguished

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since each has unique pair of endpoints in S. Thus $em\dim(G) = 4 = n$.
Case 3: $n \geq 5$. Let $S = \{u_1, u_2, \dots, u_{n-1}\}$. We prove S resolves all edges.

(i) Edge pendant and pendant. For $e_i = vu_i$, $e_j = vu_j$, $i \neq j$: $d_m(u_i, e_i) = 0$ and $d_m(u_i, e_j) = d(u_i, u_j) \geq 1$. Distinct.

(ii) Edge cycle and cycle. For distinct cycle edges f_i, f_j: Choose $s = u_i \in S$. $d_m(u_i, f_i) = 0$ and $d_m(u_i, f_j) \geq 1$. Distinct.

(iii) Edge pendant and cycle. Compare $e_i = vu_i$ and $f_j = u_j u_{j+1}$. If $i \neq j, j+1$, $d_m(u_i, e_i) = 0$ and $d_m(u_i, f_j) \geq 1$. If $i = j$, then both distances from u_i equal 0, but since $n \geq 5$, there exists another vertex $u_k \in S$ breaking equality (cycle no longer has global symmetry as in $n = 4$).

Hence all edges are distinguished. Thus, $em\dim(G) \leq n - 1$. Assume $|S| \leq n - 2$. Then at least two cycle vertices u_i, u_j are not in S. For $n \geq 5$, the cycle contains two vertices with identical distance vectors to S (due to cyclic symmetry when two are omitted). Hence pendant edges vu_i and vu_j have identical representations, contradiction. Thus $|S| \geq n - 1$.
Then we can conclude

$$em\dim(K_1 \odot C_n) = \begin{cases} n & , n \in \{3,4\} \\ n-1 & , n \geq 5 \end{cases}$$

■

4. Conclusions

In this paper, we introduced the concept of the edge monophonic metric dimension as an extension of both the edge metric dimension and the monophonic metric dimension. By defining the monophonic distance from a vertex to an edge, we established the notion of an edge monophonic resolving set and the corresponding edge monophonic metric dimension. We first established fundamental properties of edge monophonic resolving sets and showed that the vertex set of every connected simple graph is an edge monophonic resolving set. We then determined the edge monophonic metric dimension for path and cycle graphs. We obtained $em\dim(P_n) = 1$, $n \geq 3$ and $em\dim(C_n) = 2$, $n \geq 3$.

Furthermore, we extended the investigation to corona product graphs and obtained exact values for $P_n \odot K_1$, $C_n \odot K_1$, $K_1 \odot P_n$, and $K_1 \odot C_n$. Specifically,

$$em\dim(P_n \odot K_1) = 2, \quad n \geq 3$$

While

$$em\dim(C_n \odot K_1) = \begin{cases} 2 & , n = 3 \text{ or } n \geq 4 \text{ } even \\ 3 & , n \geq 4 \text{ } odd \end{cases}$$

and

$$em\dim(K_1 \odot P_n) = \begin{cases} n & , n = 3 \\ n-1 & , n \geq 4 \end{cases}$$

as well as

$$em\dim(K_1 \odot C_n) = \begin{cases} n & , n \in \{3,4\} \\ n-1 & , n \geq 5 \end{cases}$$

These results demonstrate that the edge monophonic metric dimension provides a new perspective on distance-based graph parameters by combining edge-based representations with monophonic distance. The results obtained in this study provide a foundation for further investigation of this parameter. Future

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research may focus on determining the monophonic metric dimension for broader classes of graphs and other graph operations, investigating its computational complexity, and exploring potential applications in network identification and structural analysis.

Author Contributions

Ana Diana Silvana Bimolo conceived the research idea and developed the overall research framework. She formulated the research objectives and questions, designed the methodology, and determined the approaches used to investigate the problem. She carried out formal analysis, including the derivation, verification, and interpretation of the mathematical results. She also prepared the initial manuscript by organizing the theoretical development, presenting the results, and writing the original draft.

Rinurwati supervised the research process and provided guidance throughout the development of the study. She contributed to validating the research methodology, mathematical arguments, and obtained results, ensuring their consistency and correctness. She also critically reviewed the manuscript, provided substantive comments and suggestions for improvement, and contributed to revising and editing the manuscript to strengthen its clarity, coherence, accuracy, and overall presentation. Both authors discussed the results and approved the final version of the manuscript.

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Declaration of Competing Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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