

Dynamical Analysis and Optimal Control of SVIR Hepatitis B Model Spread in Malang

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ARTICLE INFO	ABSTRACT
<p><u>Article History</u> Received : 28 Nov 2022 Revised : 02 Jan 2023 Accepted : 14 Aug 2023 Available Online : 30 Aug 2023</p> <hr/> <p>Keywords: Stability Analysis Optimal Control HBV Epidemic Model Saturated Incidence Rate Numerical Simulation</p> <hr/> <p>Please cite this article APA style as: Drajat, P. P., Cipta, B. S. I., Nuruddin, I. F., & Balqis, D. A. I. (2023). Dynamical Analysis and Optimal Control of SVIR Hepatitis B Model Spread in Malang. <i>Vygotksy: Jurnal Pendidikan Matematika dan Matematika</i>, 5(2), pp. 77-90.</p>	<p>This research formulates a new mathematical model that describes the dynamics of the spread and control of Hepatitis B, especially in Malang, East Java. The mathematical model that will be constructed considers various factors in Malang, mainly the saturated Incidence rate and vaccinations carried out on newborns. The equilibrium point of the system is determined and its stability is dynamically analyzed. The results of this analysis will determine the future prediction of the spread of Hepatitis B in Malang. Furthermore, a control strategy is proposed that is Clean and Healthy Living behavior (PHBS). The control strategy is analysed using optimal control theory. Pontryagin’s principle is used as a condition to obtain optimal conditions. Forward-backwards Sweep method in combination with the fourth order Runge-Kutta method is used to solve the optimal system.</p>

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1. Introduction

Hepatitis B is an infectious liver inflammatory disease caused by infection with the hepatitis B virus (HBV). Hepatitis B can cause acute illness and chronic conditions leading to cirrhosis and liver cancer. The process of transmission of the hepatitis virus can be through sexual intercourse, using unsterilized needles, blood transfusions, and other body fluids such as sperm, vaginal fluids, breast milk, tears, saliva, and liquids in open canker sores that have been infected. This disease can also be transmitted vertically from mother to fetus in the womb (S. Tsukuda, 2020).

Hepatitis B is a deadly disease that is a global problem. More than two billion individuals have been infected with Hepatitis B, and 80% of primary liver cancer patients are also caused by this Hepatitis B infection (Kesehatan, 2012). In Indonesia, based on the data and information center of the Indonesian Ministry of Health, it is estimated that there are twenty eight million Indonesians infected with hepatitis B or C, 14 million of whom have the potential to get chronic hepatitis B disease (Kesehatan, Situasi dan Analisis Hepatitis, INFODATIN: Pusat Data dan Informasi Kementerian Kesehatan RI, 2014). In Malang, based on data from RSUD Dr. Saiful Anwar Malang, The number of individuals infected with Hepatitis B in 2019 was 126 individuals. This number increased by 3.28% from the previous year, which reached 122 individuals.

The government has carried out various policies to control the spread of hepatitis B, one of which is immunization. Immunization of newborns in Indonesia is carried out in 4 stages in the age range of 0-4 months. In 2016 the coverage of Hb-0 immunization was 96.71% (Pertiwi, 2020). However, the incidence of hepatitis B infection was still high. Based on the data, the number of infections each year in 2015-2019 was 105, 81, 137, 122, 126 individuals (Aulani, Kusumawinahyu, & Darti, 2021).

Various studies have been conducted to control the spread of Hepatitis B, as in (Golgeli, 2019) (Khan, Zaman, & Chohan, 2018). The mathematical model of hepatitis B spread with saturation infection rate was constructed (Khan, Ullah, Ali, & Zaman, 2019). Three compartments are used: Susceptible Individuals, Infected Individuals, and Recovered Individuals. A dynamic analysis of the solutions obtained has been carried out to determine the characteristics of the spread in endemic conditions and disease-free conditions. Control strategies are given in prevention education, treatment, and vaccination. Furthermore, a large-scale simulation is carried out from the analysis results.

The estimation of the parameters that affect the spread of Hepatitis B in Malang has been carried out (Aulani, Kusumawinahyu, & Darti, 2021). This study developed the model from (Khan, Ullah, Ali, & Zaman, 2019), which uses the SIR compartment with eight parameters. Based on data from Dispendukcapil Malang and RSUD Dr. Saiful Anwar in 2015-2019, 3 fixed parameters were obtained, and five other parameters were estimated. These parameters will be used in the model so that the model solution is determined according to the conditions in Malang.

Facts reveal that vaccination for newborns in Malang reached more than 90% in 2016. However, the incidence of Hepatitis B was still high from 2015-2019. This is because the generation without vaccination is still more than the generation that has been vaccinated. Therefore, it is necessary to use a model that better describes this situation. The SIR model is refined by adding a new compartment, namely V, which states the number of individuals vaccinated as (Djilali & Bentout, 2021) (Oke, Ogunmiloro, Akinwumi, & Raji, 2019) (Zhang & Upadhyay, 2021). (Oke, Ogunmiloro, Akinwumi, & Raji, 2019) discuss the deterministic model of SIRV using the assumption that the vaccinated individual will lose immunity and become susceptible again after some time.

In this research, the model in (Khan, Ullah, Ali, & Zaman, 2019) was collaborated with the SIRV epidemic model in (Oke, Ogunmiloro, Akinwumi, & Raji, 2019). So that four compartments were formed, namely $S(t)$ the number of healthy susceptible individuals, $V(t)$ the number of individuals who had been vaccinated, $I(t)$ the number of infected individuals, and $R(t)$ the number of

recovered individuals. Then used, the parameter estimation was conducted by Aulani (2021) So it is hoped that the model formed, in addition to the characteristics of the spread of Hepatitis B, can also describe the actual conditions in Malang. Furthermore, a control strategy is proposed in the form of PHBS to the community. The process is analyzed using optimal control theory, Pontryagin's principle is used as a condition to obtain optimal conditions. The solution to the optimal system is using the MATLAB-assisted sweep method (Darajat, Suryanto, & Widodo, 2016) (Darajat & Husadaningsih, 2019). This optimal control stage results in the amount of effort to control PHBS that needs to be done and its effectiveness in preventing the spread of Hepatitis B, especially in Malang.

2. Method

Two theories of analysis will be used, namely Dynamic Analysis and Optimal Control. In both analyzes, simulations were carried out at a specific population scale to obtain the characteristics of the spread and control of Hepatitis B in the future and the effect of the PHBS strategy on the model. Determination of solutions and system simulations are used with the help of MATLAB software. The stages of the research are as follows:

- a. Literature study was conducted on the characteristics of hepatitis B transmission, mathematical models constructed in previous studies, and data on hepatitis B infection in Malang at 2015-2019.
- b. A deterministic mathematical model is constructed that describes the spread of Hepatitis B, defines the compartments used, is symbolized by influential parameters, and assumptions were built to limit the research. The model is made in the form of a system of differential equations.
- c. The model is solved using dynamic analysis theory. The equilibrium points of the model are determined. These equilibrium points describe the behavior of the system. This dynamic analysis will do several things in Existence Analysis and Equilibrium Point Stability Analysis (Alligood, Sauer, & Yorke, 2000) (Murray, 2002).
- d. The following model is written in the MATLAB source code to describe the system's behavior from time to time to determine the population changes of each compartment. Simulations are carried out using specific initial values with estimated parameter values.
- e. Based on the dynamic analysis and simulation results, interpretation is made of the characteristics of the spread of Hepatitis B in Malang.
- f. Added PHBS control strategy as a control variable to the model.
- g. The optimal control problem is defined by establishing an objective function and boundary conditions.
- h. Using the Pontryagin minimum principle, the optimal system solution is determined.
- i. The solution to the optimal system solution is written in the MATLAB source code and then re-simulated.
- j. The simulation results will be compared with the previously simulated uncontrolled system.
- k. An interpretation of the effect of PHBS control on the model is carried out.

3. Results and Discussion

3.1. Model Construction

The proposed model consists of four compartments ie. $S(t), V(t), I(t), R(t)$ which the number of healthy susceptible individuals, the number of individuals who have been vaccinated, the number of infected individuals, and the number of recovered individuals. Some of the assumptions applied to the model include:

- Individuals born into the susceptible population.
- Susceptible individuals will be infected by infected individuals with a saturated infection rate.
- Vaccines have limited effectiveness, so the vaccinated individual will revert to being a susceptible individual again.
- The population of recovered individuals has permanent immunity.

The parameters used in the system are defined as follows:

Table 1. Parameter Definition

Symbol	Definition
Λ	Birth Rate
γ	Saturation Rate
α	Infection rate
μ_0	Natural Death Rate
μ_1	Death Rate due to Hepatitis B
θ	Vaccination Rate
η	Lose Immunity Rate
β	Recovery Rate

The relationship between compartments is explained in Figure 1. as follows:

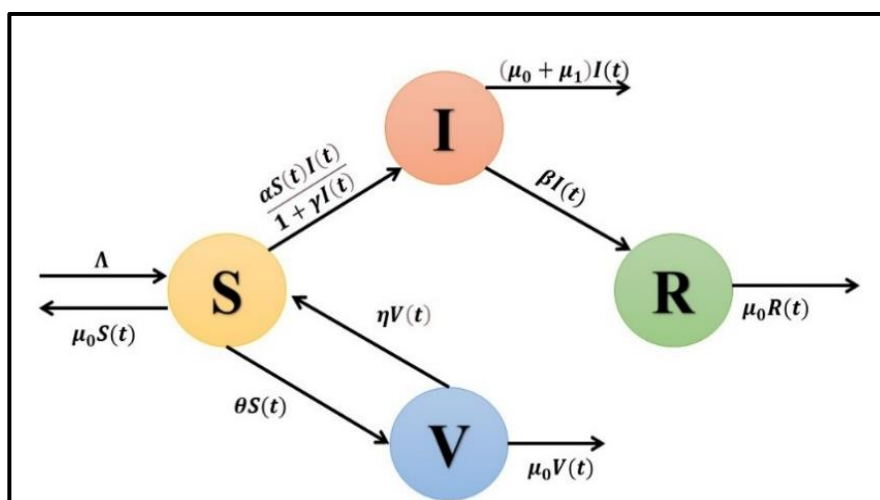


Figure 1. Compartment Diagram of Hepatitis B Spread and Control Model in Malang

Thus, a mathematical model of the spread and control of Hepatitis B in Malang can be constructed as follows:

$$\begin{aligned} \frac{dS(t)}{dt} &= \Lambda - \frac{\alpha S(t)I(t)}{1 + \gamma I(t)} - \mu_0 S(t) - \theta S(t) + \eta V(t) \\ \frac{dV(t)}{dt} &= \theta S(t) - \eta V - \mu_0 V(t) \\ \frac{dI(t)}{dt} &= \frac{\alpha S(t)I(t)}{1 + \gamma I(t)} - (\mu_0 + \mu_1 + \beta)I(t) \\ \frac{dR(t)}{dt} &= \beta I(t) - \mu_0 R(t) \end{aligned} \tag{1}$$

3.2. Equilibrium Points

The equilibrium point of the model is obtained when the steady state, i.e. satisfies the following equation,

$$\frac{dS(t)}{dt} = \frac{dV(t)}{dt} = \frac{dI(t)}{dt} = \frac{dR(t)}{dt} = 0$$

In this situation, two equilibrium points E_1 and E_2 are found: the disease-free equilibrium point and the endemic equilibrium point. The equilibrium point E_1 is as follows.

$$E_1 = (S(t)^0, V(t)^0, I(t)^0, R(t)^0) = \left(\frac{\Lambda A}{\mu_0 B}, \frac{\Lambda \theta}{\mu_0 B}, 0, 0 \right). \tag{2}$$

Meanwhile, the equilibrium point $E_2 = (S(t)^*, V(t)^*, I(t)^*, R(t)^*)$ satisfies the following equation,

$$S(t)^* = \frac{A(\Lambda \gamma + C)}{\alpha A + \gamma \mu_0 B} \tag{3}$$

$$V(t)^* = \frac{\theta(\Lambda \gamma + C)}{\alpha A + \gamma \mu_0 B} \tag{4}$$

$$I(t)^* = \frac{\Lambda \alpha A - \mu_0 C B}{\gamma \mu_0 B C + \alpha A C} \tag{5}$$

$$R(t)^* = \frac{\beta(\Lambda \alpha A - \mu_0 C B)}{\mu_0(\gamma \mu_0 B C + \alpha A C)} \tag{6}$$

As A, B, and C are fulfilling the following equation,

$$A = \eta + \mu_0 \tag{7}$$

$$B = \eta + \mu_0 + \theta \tag{8}$$

$$C = \beta + \mu_0 + \mu_1 \tag{9}$$

Equilibrium point E_1 exists because it satisfies $S(t)^0, V(t)^0, I(t)^0, R(t)^0$ are non-negative. The equilibrium point E_2 requires conditions to exist. $S(t)^*$ and $V(t)^*$ have non-negative values. The condition that $I(t)^*$ and $R(t)^*$ are nonnegative is,

$$\frac{\Lambda \alpha A}{\mu_0 C B} > 1 \tag{10}$$

This inequality (10) is called the basic reproduction number (R_0).

3.3 Stability Analysis of Equilibrium Point

The stability of the equilibrium point is determined by analyzing the eigenvalues of the Jacobi matrix. The Jacobi matrix of system (1) is,

$$J = \begin{bmatrix} -\frac{\alpha I(t)}{1 + \gamma I(t)} - \mu_0 - \theta & \eta & -\frac{\alpha S(t)}{1 + \gamma I(t)} + \frac{\alpha S(t)I(t)}{(1 + \gamma I(t))^2} & 0 \\ \theta & -\eta - \mu_0 & 0 & 0 \\ \frac{\alpha I(t)}{1 + \gamma I(t)} & 0 & \frac{\alpha S(t)}{1 + \gamma I(t)} - \frac{\alpha S(t)I(t)\gamma}{(1 + \gamma I(t))^2} - C & 0 \\ 0 & 0 & \beta & -\mu \end{bmatrix} \quad (11)$$

So the obtained Jacobi matrix around E_1 is,

$$J(E_1) = \begin{bmatrix} -\mu_0 - \theta & \eta & -\frac{\alpha \Lambda A}{\mu_0 B} & 0 \\ \theta & -\eta - \mu_0 & 0 & 0 \\ 0 & 0 & \frac{\alpha \Lambda A}{\mu_0 B} - C & 0 \\ 0 & 0 & \beta & -\mu \end{bmatrix} \quad (12)$$

The eigenvalues of the matrix $J(E_1)$ are obtained by solving $|J(E_1) - \lambda I| = 0$.
 The eigenvalues are,

$$\lambda_1 = \frac{\Lambda \alpha A - \mu_0 C B}{\mu B} \quad (13)$$

$$\lambda_2 = -B \quad (14)$$

$$\lambda_3 = \lambda_4 = -\mu \quad (15)$$

Seen in (13) and (14) the three eigenvalues $\lambda_2, \lambda_3, \lambda_4$ are negative. Meanwhile, the eigenvalue λ_1 will be negative if it satisfies,

$$\frac{\Lambda \alpha A}{\mu_0 C B} < 1 \quad (16)$$

Based on equations (10), (13), (14), and (15), it can be seen that when the equilibrium point E_1 is stable, the equilibrium point E_2 does not exist. Furthermore, the Jacobi matrix around the equilibrium point E_2 is as follows,

$$J = \begin{bmatrix} -\frac{\Lambda \alpha A + \mu_0 B C}{AC + \gamma \Lambda A} - \mu_0 - \theta & \eta & C \left(-1 + \frac{\Lambda \alpha A - \mu_0 B C}{\alpha AC + \alpha \gamma \Lambda A} \right) & 0 \\ \theta & -\eta - \mu_0 & 0 & 0 \\ \frac{\Lambda \alpha A - \mu_0 B C}{AC + \gamma \Lambda A} & 0 & C \frac{\Lambda \alpha A - \mu_0 B C}{\alpha AC + \alpha \gamma \Lambda A} & 0 \\ 0 & 0 & \beta & -\mu \end{bmatrix} \quad (17)$$

The characteristic polynomial of the Jacobi matrix in (17) is.

$$(\lambda + \mu)(\lambda^3 + K_1 \lambda^2 + K_2 \lambda + K_3) = 0 \quad (18)$$

With the values of K_1, K_2, K_3 satisfy,

$$K_1 = x + y + z \quad (19)$$

$$K_2 = xy + xz + yz - \eta\theta - \alpha z + \frac{\alpha z^2}{C} \quad (20)$$

$$K_3 = xyz - \eta\theta z - \alpha z + \frac{\alpha z^2}{C} \quad (21)$$

$$\text{Meanwhile, } x, y, z \text{ is satisfy,} \quad (22)$$

$$x = \frac{\Lambda\alpha A - \mu_0 BC}{AC + \gamma\Lambda A} + \mu_0 + \theta$$

$$y = \eta + \mu$$

$$z = -C \frac{\Lambda\alpha A - \mu_0 BC}{\alpha AC + \alpha\gamma\Lambda A}$$

It was found that $\lambda_1 = -\mu$ while λ_2, λ_3 and λ_4 were determined using the Routh-Hurwitz criteria. Equilibrium point E_2 will be locally asymptotically stable if it satisfies $R_0 > 1, K_1, K_2, K_3 > 0$, and $K_1 K_2 > K_3$.

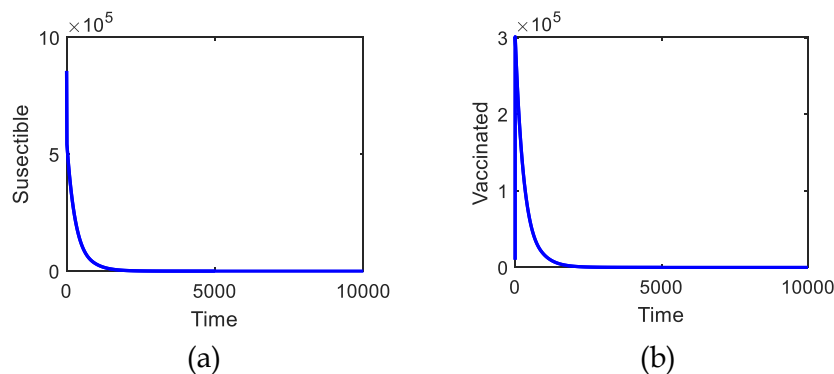
3.4 Numerical Simulation of The Model

Numerical simulation of the Mathematical model of the spread and control of Hepatitis B in (1) was carried out to support the dynamic analysis results in section 3.3. The simulation was carried out using the fourth order Runge-Kutta method, assisted by MATLAB software. Parameters and initial values are used to refer to (Aulani, Kusumawinahyu, & Darti, 2021). Because the total population in Malang is 886.801 people, the initial population assumed is, $(S_0, V_0, I_0, R_0) = (856.774, 10.000, 27, 0)$. Meanwhile, the parameter values used are presented in the following table.

Table 2. Parameter Value

Symbol	Definition	Value	Unit
Λ	Birth Rate	0,0025128	Per day
γ	Saturated Rate	0,022	Per day
α	Infection Rate	0,0000311	Per day
μ_0	Natural Death Rate	0,0024205	Per day
μ_1	Death Rate Due to Hepatitis B	0,0000018	Per day
θ	Vaccination Rate	0,965	Per day
η	Imunity Loss Rate	1,788	Per day
β	Recovered Rate	1,248	Per day

Based on the parameters in Table 2 is obtained $R_0 = 1.67 \times 10^{-5} < 1$ so satisfies the stability requirements of the equilibrium point E_1 . The simulation results are presented in Figure 2.



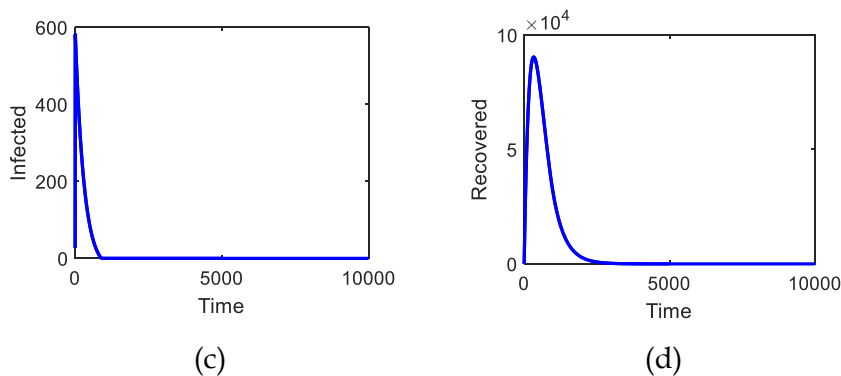


Figure 2. Simulation Result (a) Susceptible Population (b) Vaccinated Population (c) Infected Population (d) Recovered Population.

Based on the simulation results in Figure 2. It is known that at time $T=10.000$, the solution reaches the equilibrium point E_1 at (2). Solution towards the point $(S_T, V_T, I_T, R_T) = (0,6746, 0,3636, 0, 0)$. The population of susceptible individuals continues to decrease as the number of vaccinated and infected individuals increases. Although a disease-free state was achieved at one time, the highest population of infected individuals reached 581. This is certainly not desirable. Therefore it is necessary to add other ways to control the number of infected individuals. Control of the spread of hepatitis B in Malang will then be carried out using the Optimal Control Theory described in the following subchapter.

3.5. Optimal Control

It is defined that $u(t)$ is a control variable that states efforts to prevent hepatitis B transmission through PHBS education. In this case, $u(t)$ represents the normalized value such that the control limit is given on set U as follows,

$$U = \{u(t) \in L: 0 \leq u(t) \leq 1, t \in [0, T]\} \tag{23}$$

Through $u(t)$ control, the transmission process due to interactions with infected individuals will be minimized. The following is a model of the spread of Hepatitis B with controls.

$$\begin{aligned} \frac{dS(t)}{dt} &= f_1(S, V, I, R, u) \\ &= \Lambda - \frac{\alpha S(t)I(t)}{1 + \gamma I(t)}(1 - u(t)) - \mu_0 S(t) - \theta S(t) + \eta V(t) \\ \frac{dV(t)}{dt} &= f_2(S, V, I, R, u) = \theta S(t) - \eta V - \mu_0 V(t) \\ \frac{dI(t)}{dt} &= f_3(S, V, I, R, u) = \frac{\alpha S(t)I(t)}{1 + \gamma I(t)}(1 - u(t)) - (\mu_0 + \mu_1 + \beta)I(t) \\ \frac{dR(t)}{dt} &= f_4(S, V, I, R, u) = \beta I(t) - \mu_0 R(t) \end{aligned} \tag{24}$$

In the optimal control problem, the variables contained in the system of differential equations are called state variables. These state variables will be controlled by being treated in the form of an appropriate control function. The control function in question can affect the behavior of the state in such a way that it can optimize the objective functional in the form of an integral function that depends on the state variable and control variable. The primary technique for

solving the optimal control problem is determining the necessary conditions that satisfy the optimal conditions and states. The necessary condition proposed by Pontryagin in 1950 is called an adjoint function. The adjoint function is used to add differential equations to the objective functional. The adjoint function can be generated by the Hamilton function (Lenhart & Workman, 2015). Pontryagin's principle is used to determine optimal condition. Then, Forward-Backward Sweep method combined with fourth-order Runge-Kutta is used to solve the optimal system. An objective functional is formed in the period $[0, T]$, which describes the control objectives: minimizing the number of infected subpopulations and the costs of PHBS education. The objective function to be minimized is,

$$J(u(t)) = \int_0^T [L_1 I(t) + L_2 u^2(t)] dt \quad (25)$$

with the constraints on equation (24).

The state variables in this problem are $S(t)$, $V(t)$, $I(t)$, and $R(t)$. The corresponding adjoint variables for the state variables are defined as $\sigma_1(t)$, $\sigma_2(t)$, $\sigma_3(t)$, and $\sigma_4(t)$. Furthermore, the Hamilton function can be formed as follows,

$$H(S, V, I, R, u, \sigma_1, \sigma_2, \sigma_3, \sigma_4, t) = L_1 I(t) + L_2 u^2(t) + \sum_{i=1}^4 \sigma_i(t) f_i(S, V, I, R, u), \quad (26)$$

where the transverse boundary condition is

$$\sigma_i(T) = 0, \quad i = 1, 2, 3, 4. \quad (27)$$

The solution of the optimal control problem will be presented in the following theorem,

Theorem 1. In the optimal control problem equation (25) with the system constraint equation (24), there is an optimal control $u(t)$ and an optimal state solution $S^*(t), V^*(t), I^*(t), R^*(t)$ such that the value of $J(u^*)$ is minimum. In addition, there are adjoint variables $\sigma_1, \sigma_2, \sigma_3$, and σ_4 which meet the following equation,

$$\begin{aligned} \frac{\partial \sigma_1}{\partial t} &= \sigma_1 \left(\frac{\alpha I}{1 + \gamma I} (1 - u(t)) + \mu_0 + \theta \right) - \sigma_2 \theta - \sigma_3 \left(\frac{\alpha I}{1 + \gamma I} (1 - u(t)) \right) \\ \frac{\partial \sigma_2}{\partial t} &= -\sigma_1 \eta + \sigma_2 (\eta + \mu_0) \\ \frac{\partial \sigma_3}{\partial t} &= -L_1 + \sigma_1 (1 - u(t)) \frac{\alpha S}{(1 + \gamma I)^2} \\ &\quad - \sigma_3 \left(\frac{\alpha S}{(1 + \gamma I)^2} (1 - u(t)) - (\mu_0 + \mu_1 + \beta) \right) - \sigma_4 \beta \\ \frac{\partial \sigma_4}{\partial t} &= \sigma_4 \mu_0 \end{aligned} \quad (28)$$

Proof. The system of adjoint equations and the transversal conditions are determined through the Hamilton function in equation (26). The negative value of the derivative of the Hamilton function for each state variable is obtained below:

$$\frac{\partial \sigma_1}{\partial t} = -\frac{\partial H}{\partial S} = \sigma_1 \left(\frac{\alpha I}{1 + \gamma I} (1 - u(t)) + \mu_0 + \theta \right) - \sigma_2 \theta - \sigma_3 \left(\frac{\alpha I}{1 + \gamma I} (1 - u(t)) \right)$$

$$\begin{aligned} \frac{\partial \sigma_2}{\partial t} &= -\frac{\partial H}{\partial V} = -\sigma_1 \eta + \sigma_2 (\eta + \mu_0) \\ \frac{\partial \sigma_3}{\partial t} &= -\frac{\partial H}{\partial I} = -L_1 + \sigma_1 (1 - u(t)) \frac{\alpha S}{(1 + \gamma I)^2} \\ &\quad - \sigma_3 \left(\frac{\alpha S}{(1 + \gamma I)^2} (1 - u(t)) - (\mu_0 + \mu_1 + \beta) \right) - \sigma_4 \beta \\ \frac{\partial \sigma_4}{\partial t} &= -\frac{\partial H}{\partial R} = \sigma_4 \mu_0 \end{aligned} \tag{29}$$

The optimal condition is obtained when the Hamilton function satisfies the stationary condition, i.e

$$0 = \frac{\partial H}{\partial u^*} \tag{30}$$

Based on the stationary conditions in equation (22), it can be determined that the condition $u(t)$ to obtain the optimal system as follows,

$$0 = 2L_2 u(t) + (\sigma_1 - \sigma_3) \frac{\alpha SI}{1 + \gamma I} \tag{31}$$

Based on equations (31), the calculation of the value of $u^*(t)$ can be done with the following equation,

$$u(t) = \min \left\{ \max \left(0, \frac{(\sigma_3 - \sigma_1) \alpha SI}{2L_2(1 + \gamma I)} \right), 1 \right\} \tag{32}$$

Let S^*, V^*, I^* , and R^* be the optimal state conditions, and $\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_4^*$ are the conditions of the corresponding adjoint variables. The optimal condition is obtained by solving the system which has substituted the value of $u^*(t)$ as follows:

$$\begin{aligned} \frac{dS^*(t)}{dt} &= f_1(S^*, V^*, I^*, R^*, u^*) \\ \frac{dV^*(t)}{dt} &= f_2(S^*, V^*, I^*, R^*, u^*) \\ \frac{dI^*(t)}{dt} &= f_3(S^*, V^*, I^*, R^*, u^*) \\ \frac{dR^*(t)}{dt} &= f_4(S^*, V^*, I^*, R^*, u^*) \\ \frac{\partial \sigma_1^*}{\partial t} &= \sigma_1^* \left(\frac{\alpha I^*}{1 + \gamma I^*} (1 - u^*(t)) + \mu_0 + \theta \right) - \sigma_2^* \theta - \sigma_3^* \left(\frac{\alpha I^*}{1 + \gamma I^*} (1 - u^*(t)) \right) \\ \frac{\partial \sigma_2^*}{\partial t} &= -\sigma_1^* \eta + \sigma_2^* (\eta + \mu_0) \\ \frac{\partial \sigma_3^*}{\partial t} &= -L_1 + \sigma_1^* (1 - u^*(t)) \frac{\alpha S^*}{(1 + \gamma I^*)^2} \\ &\quad - \sigma_3^* \left(\frac{\alpha S^*}{(1 + \gamma I^*)^2} (1 - u^*(t)) - (\mu_0 + \mu_1 + \beta) \right) - \sigma_4 \beta \\ \frac{\partial \sigma_4^*}{\partial t} &= \sigma_4^* \mu_0 \end{aligned} \tag{33}$$

3.6. Model Simulation with Control

The optimal system (33) is solved numerically using the forward-backward Sweep method. As the initial values and parameters used in section 3.4, the simulation results are obtained in Figure 3. and Figure 4.

Based on Figure 3, it is known that controlling $u(t)$ does not affect the number of susceptible and Vaccinated populations. Controlling $u(t)$ significantly reduced the number of infected individuals and reduced the subpopulation of recovered individuals. Based on Figure 5. At the beginning of the control process, $u(t)$ must be carried out maximally (value 1). When the number of infected individuals decreases, the control process can be reduced to 0.

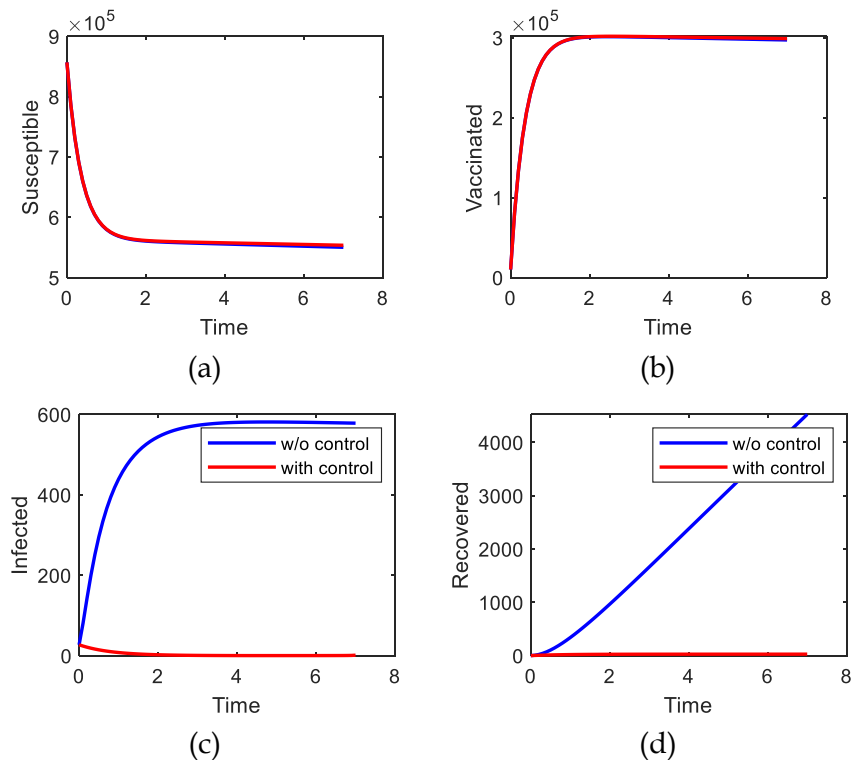


Figure 3. Simulation Result (a) Susceptible Rate with Control (b)Vaccinated rate with Control (c) Infected Rate with Control (d) Recovered Rate with Control.

The control rate $u(t)$ is presented in Figure 4.

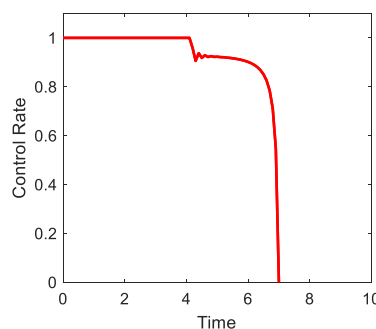


Figure 4. Control Rate $u(t)$

4. Conclusions

Based on the research's overall results it was found that,

- a. In the model, the saturation incidence rate is used. The disease-free equilibrium point was asymptotically stable when $R_0 < 1$. Meanwhile, the endemic equilibrium point exists if $R_0 > 1$ and is locally asymptotically stable if certain conditions are met. Therefore, the parameter values taken according to the conditions in Malang meet $R_0 < 1$. This means that in the future, a state of being free from hepatitis B will be achieved. However, based on the simulation results, the number of infected individuals reached 581 before finally decreasing and reaching a disease-free state.
- b. A control strategy was added in the form of education on the community's clean and healthy living behavior (PHBS). The numerical simulations found that when the educational strategy is carried out optimally, it reduces the number of infected individuals significantly.

Author Contributions

First author and second author contributed to construct the model. First author and third author contributed to theoretical analysis and the interpretation of the results. Second author and fourth author contributed to numerical analysis and simulation. First author as corresponding author revised the article based on review from reviewer and journal editor. All authors provided critical feedback and helped shape the research, analysis and manuscript.

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Declaration of Competing Interest

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