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The Effect of Nanoparticles on Drug Distribution in The Mathematical Model of Blood Flow

Yolanda Norasia [1*], Ajeng Oxa Nisa [1], Sitti Rosnafi'an Sumardi [2]

[1]Department of Mathematics, Walisongo State Islamic University, Indonesia
[2]Department of Mathematics, Cendrawasih University, Indonesia

**Email Correspondence:* yolandanorasia@walisongo.ac.id

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ABSTRACT

This research examines the influence of nanoparticles in the distribution of drugs in healthy blood flow on linear, angular velocity and blood temperature. Construction and simplification of a blood flow model based on boundary layer equations, dimensionless variables, stream functions, and similarity variables. The initial step is to establish a dimensional blood flow model. Using dimensionless variables, the equation is simplified into a dimensionless equation. A similarity equation is generated by converting the non-dimensional equation. The nanoparticles used are Cu, TiO_2, Al_2O_3. At the linear velocity and temperature of blood flow, Al_2O_3 is in the highest position. At the angular velocity of blood flow, the position of blood flow with Cu nanoparticles is in the uppermost position. This research is used to estimate the velocity and temperature of blood flow with the influence of nanoparticles as drug distribution.

1. Introduction

Mathematical modeling is part of applied mathematics. The formulation of mathematical models is used to find a solution to a problem, and can be applied in the applied sciences of physics, biology, and so on. An example of applying a mathematical model is fluid flow. The viscosity classification of fluids is divided into two, namely viscous fluids and inviscid fluids (Norasia et al., 2022). In terms of viscous fluids, blood falls into this category. Blood is in the human circulatory system and plays an important role in the internal transportation of the human body. Blood flow moves throughout the body to distribute oxygen and nutrients to all cells and body tissues. The blood flow media are blood vessels, namely arteries, veins, and capillaries. The blood flow velocity in the middle and edges

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of the artery is the same in laminar flow(Saqr et al., 2020). Mathematical models regarding bone tissue blood flow are solved using computational fluid dynamics (D. Ali & Sen, 2018). Blood flow with constant pressure boundary conditions at the coronary ostium is solved using computational fluid dynamics (Yoshikawa et al., 2020). Modeling blood flow in viscoelastic vessels can be used to model human circulation in arteries and veins (Bertaglia et al., 2020). Mathematical modeling of blood flow through three different types of stenosis shows nonnewtonian behavior can influence blood flow behavior (Owasit & Sriyab, 2021). Mathematical models of blood flow with hematological disorders show that changes in hematocrit values can significantly influence the physical quantity and volumetric flow rate(Karthik et al., 2022). Mathematical models of blood flow in stenosis show that velocity increases as the angle of inclination increases (Dhange et al., 2022). The Navier-Stokes equations in blood flow can be solved using the finite difference method (Hu et al., 2023).

Nanotechnology is the science that studies the manipulation of materials on the atomic and molecular scale. Nanoparticles are one of the main applications of nanotechnology. Nanoparticles have sizes between one and 100 nanometers. The physical and chemical properties in the form of electronic, magnetic and thermal stability of nanoparticles make them superior to large-sized materials (Fahmi, 2020). Nanoparticle size is used to control particle size, surface properties and release of active substances in drug delivery systems. Nanoparticles added to the base fluid are called nanofluids. Al_2O_3 nanoparticles can increase friction by 67 percen (Wei et al., 2021). The influence of the Eckert number and nanoparticles causes the fluid temperature to increase (Gul et al., 2021). Nanofluid research carried out by comparing metal particles and metal oxides shows that metal oxides move faster than metal particles (Norasia et al., 2023).

In the medical field, nanoparticles are widely used for diagnostic and therapeutic purposes. Gold (Au) nanoparticles can be used for drug delivery, cancer detection, and photothermal therapy (Singh et al., 2018). The effect of Cu, TiO_2, Al_2O_3 particles can increase blood flow velocity in blocked blood vessels (Zaman et al., 2019). Mathematical models of blood flow show that the length of blood vessels can influence blood pressure (Khalid et al., 2021). Blood flow with nanoparticle suspension through blood clot arteries shows that as the size of the clot increases, the blood temperature also increases (Shah & Kumar, 2020). The application of nanoparticles is important for further development. The ability of nanoparticles as drug distribution is interesting to research. Treatment efficacy is increased by targeting diseased cells and avoiding healthy cells with nanoparticles (Yusuf et al., 2023). This research aims to look at the linear velocity, angular velocity, temperature and pressure of blood flow with the addition of particles Cu, TiO_2, Al_2O_3. The blood flow medium is a porous artery without any blockages. The model building process starts from dimensional similarities, non-dimensional similarities, and similarities. The similarity equation is solved using the Thomas and Backward Euler Algorithms.

2. Method

This research involves physics and mathematics approaches in building a blood flow model, so there are research stages as follows.

2.1. Mathematical Model Construction

In this research, a mathematical model was built including continuity,

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momentum, and energy equations. The model equation obtained is then simplified into a non-dimensional equation with non-dimensional variables. The non-dimensional equation obtained was substituted for the nanoparticle and base fluid variables. In this research, nanoparticles and blood as base fluid were used with the following parameter values.

Table 1. Nanoparticles and Basic Fluid Parameters/Blood (Ahmed & Nadeem, 2016) (Ali et al., 2022)

Parameters	Al_2O_3	TiO_2	Cu	Blood
Density (ρ)	3970	4250	8933	1063
Spesific Heat (c_p)	765	686.2	385	3594
Conductivity (k)	40	8.9538	400	0.492

Taking the boundary layer approach, using the flow function, and using the similarity variables, the equation becomes one variable (similarity). The following is the equation for the boundary layer approach, flow function, and similarity variable.

Boundary layer equation (Saeed et al., 2021)

$$Re \to \infty, Re = \frac{r\alpha u}{v_f} \tag{1}$$

with
Re is Reynolds number

Stream function (Ullah et al., 2020)

$$u(x,y) = \frac{\partial \psi}{\partial y}, v(x,y) = -\frac{\partial \psi}{\partial x} \tag{2}$$

with
ψ is stream function variables
u, v is velocity of blood flow

Similarity Variables (Alsenafi & Ferdows, 2022)

$$\psi = F(x, Y, t)u\,(x)r(x) \tag{3}$$

with
u_e is velocity of free flow
r is radius of the porous medium

3. Results and Discussion

This research discusses the development of a blood flow model with the addition of nanoparticles for drug distribution. By Building continuity, momentum, and energy equations by applying the laws of physics of Lavoisier, Newton, and thermodynamics. The addition of nanoparticles in the bloodstream is measured through density ρ_{fn} and viscosity μ_{fn}. Then the microrotation ability of the blood flow is measured using the microrotation parameter N. The mathematical model of blood flow forms four, the first is the continuity equation, namely the rate of mass change is constant. Second, the linear momentum equation which is influenced by pressure p, magnetic parameters B, and gravity on the x-axis g_x

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and y-axis g_y. Third is the angular momentum equation, namely the existence of microrotation in the blood which is influenced by γ , $\mathcal{I}$. Fourth, the energy equation is influenced by heat diffusivity α to temperature T of blood flow. The dimensional equations of continuity, momentum, and energy are respectively obtained as follows.

1. $\frac{\partial ru}{\partial x} + \frac{\partial rv}{\partial y} = 0$

2. $\rho_{fn}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left(\mu_{fn} + k\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \sigma B^2 u - \rho_{fn}(T - T_\infty)\, g_x + k\frac{\partial N}{\partial y}$

3. $\rho_{fn}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \left(\mu_{fn} + k\right)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \sigma B^2 v - \rho_{fn}(T - T_\infty)\, g_y + k\frac{\partial N}{\partial x}$

4. $\rho_{fn}\,\mathcal{I}\left(\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) - k\left(2N - \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

5. $\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$

(4)

The boundary conditions of the model are.

$t = 0, u = v = 0, N = 0, T = T_\infty$ when x, y

$t > 0, u = v = 0, T = T_w, N = -N\frac{\partial u}{\partial t}$ when $y = 0$

$u = u_e(x), u = v, N = 0, T = T_\infty$ when $y \to \infty$

The next step is to approach the boundary layer (1), flow function (2) and similarity variable (3), equation (4) becomes the similarity equation for linear, angular momentum and temperature respectively as follows.

1. $\frac{(1+K)}{\left((1-\chi)+\chi\left(\frac{\rho_s}{\rho_f}\right)\right)(1-\chi)^{2.5}}\frac{\partial^3 F}{\partial Y^3} + K\frac{\partial H}{\partial y} + \frac{3}{2}\cos x\left[-\left(\frac{\partial F}{\partial Y}\right)^2 + 2F\frac{\partial^2 F}{\partial Y^2} + 1\right] = \frac{\partial^2 F}{\partial t \partial Y} + \frac{3}{2}\sin x\left[-\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial Y^2} + \frac{\partial F}{\partial Y}\frac{\partial^2 F}{\partial x \partial Y}\right] + \left(1 - \frac{\partial F}{\partial Y}\right)M - \frac{2}{3}\lambda S$

2. $\left(1 + \frac{K}{2}\right)\frac{\partial^2 H}{\partial Y^2} + \frac{3}{2}\cos x\left(-H\frac{\partial F}{\partial Y} + 2F\frac{\partial H}{\partial Y}\right) = \frac{\partial H}{\partial t} + K\left(\frac{\partial^2 F}{\partial Y^2} + 2H\right) + \frac{3}{2}\sin x\left(\frac{\partial H}{\partial x}\frac{\partial F}{\partial Y} + \frac{\partial H}{\partial Y}\frac{\partial F}{\partial x}\right)$

3. $\left(\frac{k_s+2k_f-2\chi(k_f-k_s)}{k_s+2k_f+\chi(k_f-k_s)}\frac{1}{\left((1-\chi)+\chi\left(\frac{\rho_s}{\rho_f}\right)\right)}\right)\frac{\partial^2 S}{\partial Y^2} + 3\cos x\, Pr\, F\,\frac{\partial S}{\partial Y} = Pr\,\frac{\partial S}{\partial t} + \frac{3}{2}\sin x\, Pr\,\left(\frac{\partial S}{\partial x}\frac{\partial F}{\partial Y} - \frac{\partial S}{\partial Y}\frac{\partial F}{\partial s}\right)$

(5)

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The boundary conditions of the model are.

$F = \frac{\partial F}{\partial Y} = 0, H = -n\frac{\partial^2 F}{\partial Y^2}, S = 1$ when $Y = 0$

$\frac{\partial F}{\partial Y} = 1, H = 0, S = 0$ when $Y \to \infty$

In the similarity equation above, the Backward Euler numerical approach and the Thomas Algorithm are carried out. Then simulated linear, angular velocity and blood temperature. The aim of this research is to look at the linear, angular velocity and blood temperature when distributing drugs using nanoparticles Al_2O_3, TiO_2, Cu.

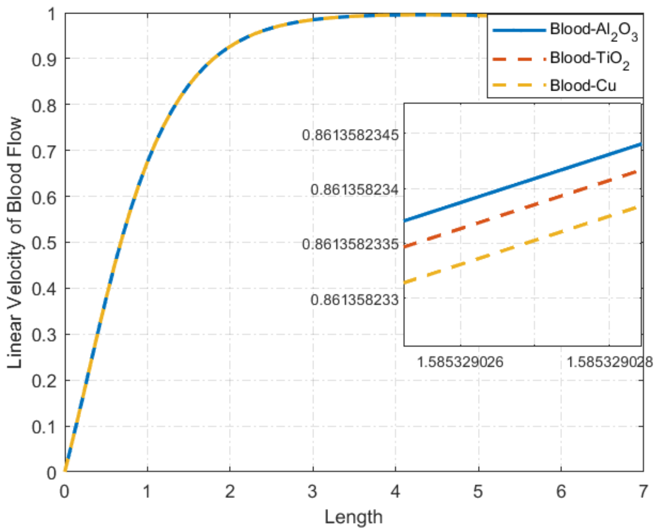

Figure 1. Linier Velocity of Blood Flow

Figure 1 shows the linear velocity of blood flow with the addition of nanoparticles Al_2O_3, TiO_2, Cu. The blue line shows blood flow with the addition of nanoparticles Al_2O_3. The red line shows blood flow with the addition of nanoparticles TiO_2. The orange line shows blood flow with the addition of nanoparticles Cu. Each nanoparticle moves at the same linear velocity, namely from zero to one. That is, the linear velocity of movement increases in the bloodstream. By enlarging the image scale at a distance between 1.585329026 to 1.585329028, it can be seen that the blue line is at the top position, in other words the linear velocity of blood flow with the addition of Al_2O_3 nanoparticles moves faster than the other two particles (TiO_2, Cu). The density of Al_2O_3 particles is 3970, which is the smallest density. Due to this, blood flow with Al_2O_3 nanoparticles is at its highest velocity.

Figure 2 shows the angular velocity of blood flow with the addition of nanoparticles Al_2O_3, TiO_2, Cu. The angular velocity of blood flow with each nanoparticle moving is the same, namely experiencing microrotational movement from zero to a peak at a distance of two and decreasing. Blood is a category of microrotational fluid, so that blood flow has an angular velocity. By enlarging the image scale to a distance of 1.9566964551, it can be seen that the orange line is at the top position, in other words the angular velocity of blood flow with the addition of Cu nanoparticles moves faster than the other two particles (TiO_2, Al_2O_3). The density of particle Cu is 8933, which is the largest density. The opposite of linear velocity, at angular velocity Cu particles move

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faster. The greater the density, the more nanoparticles. As a result, microrotation becomes greater, and the angular velocity of blood flow with Cu nanoparticles also moves greater than other particles.

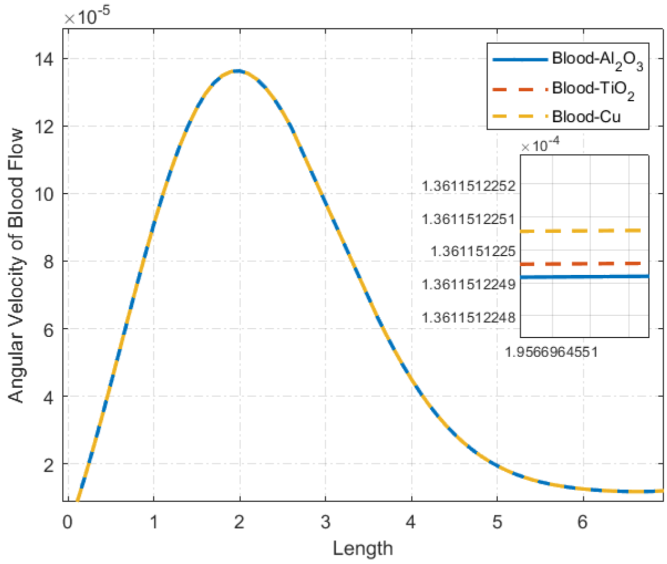

Figure 2. Angular Velocity of Blood Flow

Figure 3 shows the blood flow temperature with the addition of nanoparticles Al_{2O_3}, TiO_2, Cu. The temperature of the blood flow with each nanoparticle moves the same, namely experiencing microrotation movement from one to zero.

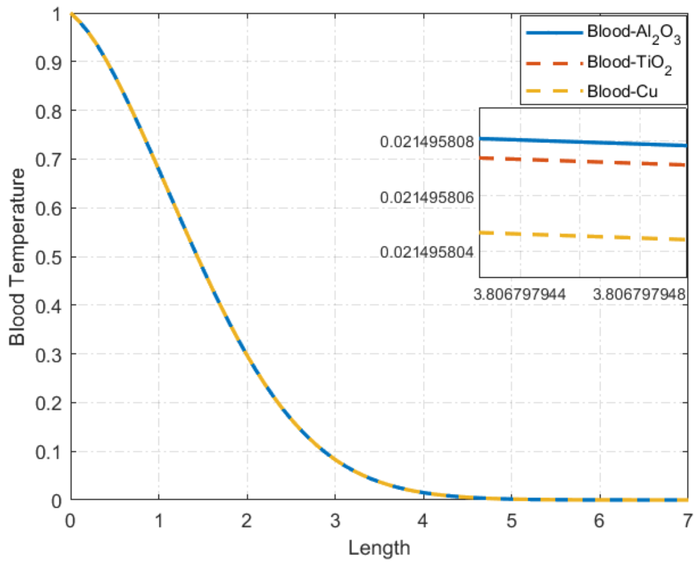

Figure 3. Blood Temperature

In other words, blood temperature decreases. By magnifying the image scale at a distance of 3.806797944, you can see the blue line at the top position, in other words the blood flow temperature with the addition of nanoparticles Al_2O_3 moves faster than the other two particles (TiO_2, Cu). Specific heat of particles Al_2O_3 is 765, which is the largest specific heat. An increase in temperature arises due to the energy that appears in a unit mass of a substance. As a result, the temperature of the blood flow with nanoparticles Al_2O_3 at highest position.

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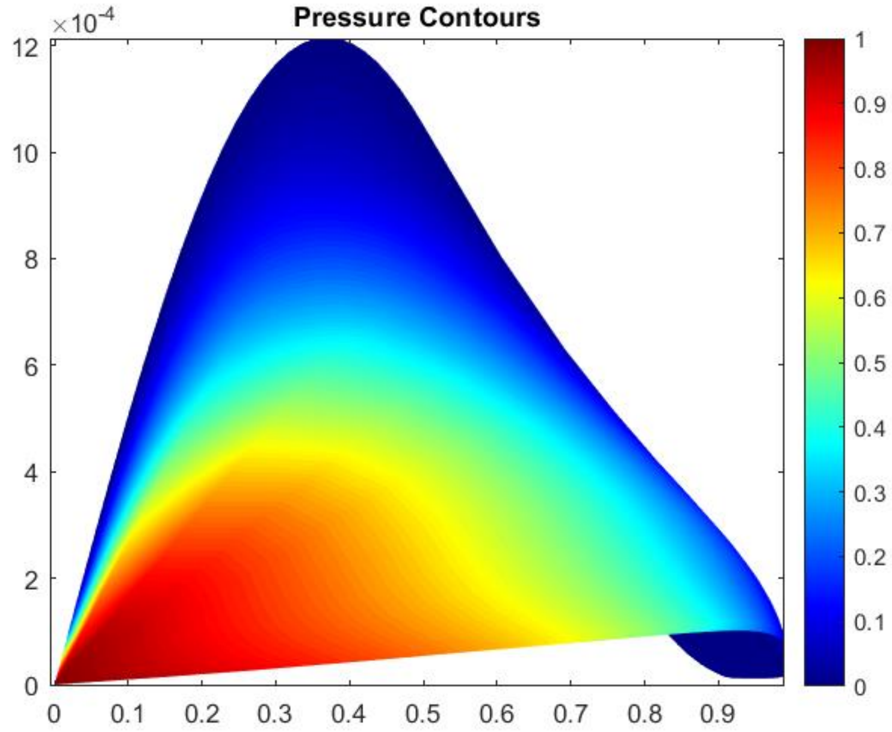

Figure 4. Pressure Blood Flow

Figure 4 shows blood flow pressure based on linear and angular velocity. Blood pressure moves from zero to a peak at a distance of 0.4 and decreases towards zero. Blood flow pressure increases at a peak of 0.4 and decreases at 0.9. Nanoparticles can interact with the blood flow to change the flow dynamics and pressure in the flow system. The more nanoparticles in the blood flow can cause an increase in pressure at certain points in the flow.

4. Conclusions

This research builds a mathematical model of blood flow by adding nanoparticles as drug distribution to the linear, angular velocity and temperature of blood flow. The model construction obtained is then simplified into non-dimensional equations and similarity equations. Numerical solution to the similarity equation uses the Backward Euler method. The simulation results show that if we want to accelerate linear velocity and increase blood flow temperature, we can use drug distribution with nanoparticles Al_2O_3. If we want to accelerate up the angular velocity, we can use nanoparticles Cu. In this case, blood flow can be modeled using the nanoparticle effect in a two-dimensional way. Blood flow can become turbulent in some areas of blood vessels. Two-dimensional models are able to properly model the turbulence phenomena that can occur in blood flow. The research can be developed in a three-dimensional environment.

Author Contributions

First author and second author contributed to construct the model and simplifying it into a similarity equation. First author and third author contribute to the solution of numerical method. All authors contributed to the analysis of

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the simulation results. Reviewers and journal editors provided feedback to the first author for the revision of the article.

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Declaration of Competing Interest

The author declares that this research has no conflicts of interest reported in this article.

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