Learning Trajectory for Learning Multiplication Rules with Islamic Values using Realistic Mathematics Education Approach

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ABSTRACT

This research aims to investigate how a learning trajectory (LT) of multiplication rules support students in developing their understanding and reasoning skills. The learning activities in LT are developed by using RME approach with Islamic values embedded. This research employed design research as a method, which involved a cyclical process of preliminary design, experimental design, and retrospective analysis. The data were collected from 30 tenth-grade students an Islamic senior high school in Jember, Indonesia. The data collected from literature reviews, interviews videotaping, observations, and tests. The result revealed that the Learning Trajectory (LT) of multiplication rules based realistic-Islamic context can support students’ thinking and reasoning about the concepts of multiplication rules. Thus, the Islamic content and context can be a basis for mathematics teacher in teaching the concept of multiplication rules.

1. Introduction

Learning permutations and combinations is similar to practicing counting principles, which are highly important for daily tasks. Humans are able to mention the number of possible occurrences or ways that an event might occur by employing the idea of this counting rule. It is essential for all teachers, in Islamic
schools especially, to begin incorporating Islamic ideals into the instructional plans they design, down to the enumeration procedures. Through education, there are many fantastic chances for pupils' character development. It may be used by educators through promoting instructional strategies built on moral character (Abdussakir, 2017). Education that incorporates values, shari'a, and religious belief is known as such by researchers. The educational process in schools is then interwoven with these principles (Nihayati, Suningsih & Abdullah, 2018).

Education reform is urgently required due to the moral degradation and absence of religious principles (Maryati, Iyam; Priatna, 2017; Rahman et al., 2015). However, this wish has not come true. With the exception of Islamic beliefs, mathematics is often taught as a single topic. Due to its rigidity, complexity, and alienation from real-world situations, mathematics study eventually turns into a topic that pupils tend to avoid. The development of Islamic values and character is less possible with mathematics. The learning achievement in mathematics is often modest, though (Salafudin, 2015).

According to Johnson (2002) and Hadi (2017) pupils will comprehend and retain what they have learnt when they discover the purpose of studying mathematics in school. Students may relate what they learn in class to actual situations they encounter every day thanks to contextual learning. Their context is expanded by contextual information. This is due to the fact that giving pupils fresh experiences might inspire their minds to form fresh connections. Consequently, students are able to discover and create meaning on their own.

The development of students’ thinking abilities requires the use of appropriate learning tactics and approaches. It is based on a commitment to improving technical and mathematical skill education (Irawan & Kencanaawaty, 2017; Sembiring et al., 2008). The reform is based on developing problem-solving skills in everyday life. Realistic Mathematics Education (RME) learning, which underpins everyday learning processes, can assist in this (Zulkardi et al., 2020). Students participate in an activity by rediscovering concepts through self-directed activities (van Galen et al., 2008; Zulkardi et al., 2020). RME emphasises real-world scenarios, which implies that students are offered visible problems (Van den Heuvel-Panhuizen & Drijvers, 2020). For problems that students can visualise and solve, the capacity to employ pedagogical tools will be required. As a result, RME is one of the variables considered as a learning approach in order for students to comprehend each activity in relation to the requirements and experiences of students related to statistics in everyday life.

Treffers (2012) proposed five principles of realistic mathematics education that provided inspiration for the process of constructing a series of learning activities in this research, starting with experience-based activities. These five elements are: employing context, progressive mathematization models, utilising pupils' own construction, interactivity, and intertwinement.

Religion and culture, according to Kuntowijoyo (2001), are two forces that interact and influence one another to produce symbols, content, and values. It was formerly stated that particular mathematical symbols have different meanings depending on religion and civilization. Therefore, it may be utilized as well in RME with the principle of real context, which includes not only concrete, physical, and tangible contexts but also what the pupil's mind can conjure up, particularly if the context spoken enters the field of religion (Abdussakir, 2017), an Islamic value-based context which pupils frequently do in everyday life.
Mathematics education that contains Islamic nuances is instruction that is based upon Islamic principles during the learning process (Isandespha, 2015). According to Salafudin (2015), learning mathematics with Islamic principles entails incorporating Islamic ideas into the material and practice problems of each class. As a result, acquiring mathematical knowledge which includes Islamic values is the same as learning mathematics which replaces Islamic values. According to Salafudin, the following values are incorporated into maths education: (1) aqidah values, or principles pertaining to things that the heart must believe is true; (2) Shari’ah principles related to a way of life as a guide to living in the world to lead to the afterlife; and (3) moral principles, i.e. values connected with an individual's soul that encourages him to behave without first thinking and considering his choices, such as morals. According to the above definition, experts are interested in doing research that attempts to enhance students' learning trajectory about the concept of multiplication rules through the use of the RME technique with Islamic principles.

In this study, utilizing a realistic mathematics education (RME) method and taking into account the challenges experienced by the students as well as the findings of the researchers stated above, we created a learning trajectory (LT) for teaching multiplication rules. A learning trajectory (LT) is a series of activities that pupils engage in to achieve a certain learning goal. (Elizondo-Ramírez & Hernández-Solis, 2017). A LT is initially created as a hypothetical learning trajectory (HLT) (Gravemeijer, Koeno; Cobb, 2013). To observe the application of learning using the RME technique, researchers created learning in the form of HLT. There are learning phases in HLT that teachers must complete in order to carry out learning. Researchers can speculate about the possible student responses using HLT. According to Bakker and Van Eerde (2013), the hypothetical learning trajectory (HLT) is the link between instructional theory and real teaching and learning. The concrete form of HLT consists of three components: learning goals for students, a series of learning activities to encourage student learning, and assumed student learning, in which the teacher anticipates the class's growth and mental processes.

A HLT eventually evolves into a theory (LT) that other math educators can use to teach a specific mathematical topic as a lesson learned. This happens following a process of developing, testing, and redesigning. While Cobb et al. (2003) called the theory a domain-specific theory, Gravemeijer, Koeno, and Cobb (2013) and Liljekvist et al. (2016) labelled it a local instructional theory (LIT). A local instruction theory consists of ideas about how to study a certain subject as well as how to assist that learning (Clements & Sarama, 2014). Scholars generated a large number of LTs in mathematics. They were utilised to teach some higher education courses, as well as mathematics in elementary and secondary schools (Afriansyah & Arwadi, 2021; Hendriana et al., 2019; Risdiyanti et al., 2019). Overall, the findings of the study demonstrated that LTs were quite effective in enhancing students' conceptual grasp.

The LT generated for this research was based on important RME principles for instructional design and served as a foundation on which the LT was carried out in the classrooms. Based on the ideas presented above, the purpose of this study is to determine the extent to which a learning trajectory of multiplication rules based on realistic Islamic principles might help pupils' thinking and understanding of multiplication rules.
2. Method
The interventionist, process-oriented, reflective elements, cyclic characters and theory-oriented design research approach was employed in this work (Nieveen, Nienke; McKenney, Susan; van den Akker, 2006; Prahmana, 2017). The approach tries to better understand how teaching and learning are interconnected in order to improve teaching (Gravemeijer, Koen; Cobb, 2013). This study was divided into three stages: preliminary design, experimental design, and retrospective analysis. Table 1 shows the research activities, which included research phases and tools.

Table 1. The research activities

<table>
<thead>
<tr>
<th>No.</th>
<th>Research stage</th>
<th>The activities</th>
<th>Data collection</th>
<th>Instruments</th>
</tr>
</thead>
</table>
| 1.  | Preliminary design   | • Reviewing literatures to gain data about students' obstacles in learning multiplication rules  
  • Designing a hypothetical learning trajectory (HLT) that includes learning techniques as well as expected student and teacher responses. | • Literatures review  
  • interview  
  • observation | • literatures  
  • interview guide sheets  
  • field notes |
| 2.  | Experimental design  | Two cycles of testing  
  • A focus group was formed to explore the feasibility of the HLT.  
  • Classroom try out: to assess the effectiveness of the HLT on students' comprehension of multiplication rules. | • Test  
  • Observation  
  • Videotaping | • written test items  
  • field notes |
| 3.  | Retrospective analysis | Revising and improving the HLT to be LT | Document analysis | |

The preliminary design resulted in a multiplication rule HLT. Six tenth grade students from an Islamic senior high school in Jember, Indonesia, were tested in small groups as part of the experimental design. The HLT was tested on 30 tenth-grade students from the same school following the processes of retrospective analysis and re-design. The retrospective analysis was carried out by the researchers, the teacher, and the observers. In the retrospective analysis, the researcher compared the assumptions and HLT with the outcomes of implementing the learning trajectory carried out at the design experiment stage,
the data gathered in the design experiment phase were analysed (Gravemeijer & Cobb, 2013). Based on the analysis, the researcher created a realistic mathematical learning trajectory of multiplication rules based on Islamic values.

3. Results and Discussion
The aim of this research is to support students in understanding the rules of enumeration of data content. The HLT for teaching multiplication rules using RME approach was created after conducting recent literature reviews to find out the need analysis, to examine 10th grade-mathematics curriculum, and to determine the students’ characteristics. The learning series consists of four activities with each context, namely (1) a map exploration for going to Masjid, (2) finding a best route for Jum’ah prayer, (3) design your map, and (4) be a good Muslim (Ningtyas et al., 2022).

<table>
<thead>
<tr>
<th>Table 2. The Learning Trajectory of Multiplication Rules</th>
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</thead>
<tbody>
<tr>
<td><strong>Activity 1. Map Exploration for Going to Masjid</strong></td>
</tr>
<tr>
<td><strong>Learning Activities</strong></td>
</tr>
<tr>
<td>1. Seeking the message of ayah about enumeration in Muslim’s life and the messages of hadist for going to Masjid.</td>
</tr>
<tr>
<td>a. Seeking the virtue QS Al-Jin: 28</td>
</tr>
<tr>
<td>b. Seeking the virtue of hadith Muslim 1009</td>
</tr>
<tr>
<td><strong>Learning Objectives</strong></td>
</tr>
<tr>
<td>Students are able to understand introduction of enumeration of an event.</td>
</tr>
<tr>
<td><strong>Students’ responses</strong></td>
</tr>
<tr>
<td>• Mention the message of ayah QS Al-Jin: 28</td>
</tr>
<tr>
<td>• Do not mention the message of ayah QS Al-Jin: 28</td>
</tr>
<tr>
<td>• Mention the virtue of hadith, i.e. every step that people take towards prayer (to Masjid) is charity (sadaqah)</td>
</tr>
<tr>
<td>• Do not mention virtue of hadith</td>
</tr>
<tr>
<td>2. Exploring a map to find the routes for going to masjid</td>
</tr>
<tr>
<td><strong>Activity 2. Finding Best Route for Jum’ah Prayer</strong></td>
</tr>
<tr>
<td><strong>Learning Activities</strong></td>
</tr>
<tr>
<td>1. Seeking the virtue of hadith of going and returning from Masjid prayers by diverse paths</td>
</tr>
<tr>
<td><strong>Learning Objectives</strong></td>
</tr>
<tr>
<td>Students can predict the probable paths of two successive events.</td>
</tr>
<tr>
<td><strong>Students’ responses</strong></td>
</tr>
<tr>
<td>• Discuss the benefits (pahala) of travelling to and from the mosque via various routes</td>
</tr>
<tr>
<td>• Do not discuss the reards/benefits (pahala) for travelling to and from the mosque via different routes</td>
</tr>
</tbody>
</table>
Learning Activities | Learning Objectives | Students’ responses
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2. Exploring the map to find possible routes for Jum’ah prayer | • Map out possible pathways with two consecutive events. | • Unable to locate the routes.

**Activity 3. Design Your Map**

1. Students find the solution in more formal way. They create a schematic in which the roads are represented by connecting lines. | Students can find possible pathways and express them more explicitly using diagrams and multiplication rules. | • State a practise for becoming a good Muslim by assisting one another
• Can construct a diagram by linking each point (homes and mosques) with lines (roads). 

2. Students understand the conclusion: To determine the number of possible ways to do something, multiply the number of options accessible at each stage (multiple rule). | • Draw lines with numbers and understand the conclusion of multiplying each number to establish travel routes. 
• Place numbers on the lines they draw, but they cannot explain how they arrived at the conclusion of multiplying each number to find the trip routes. |

**Activity 4. Be a Good Muslim**

Students determine the number of possible ways to do something by multiplying the number of options accessible at each stage (multiple rule). | Students can use multiplication rules to tackle the challenge of determining possible paths. | • Multiply the number of options available at each stage (multiple rule) 
• Do not multiply the number of options available at each stage

Two specialists in math education who are experienced with the RME approach validated the HLT. Plomp (2013) found that the HLT meets the criteria for construct and content validity. According to some standards agreed upon by the experts, the HLT met the following requirements: 1) The contextual-specific problems with Islamic values may enable the students to re-invent their understanding of multiplication rules; 2) The learning activities may assist students in experiencing the steps of horizontal and vertical mathematization. Based on the findings of the focus group experiment, the HLT for multiplication rules utilizing the RME approach could run as researcher’s expectation. It implies that the HLT met the practicality requirements (Plomp, 2013). The HLT’s promise to strengthen pupils’ capacity for reasoning when learning multiplication rules (Ningtyas et al., 2022).

The following is an example of a context problem from the HLT aimed to support students understand the multiplication rule ideas.
1) Sub-activity 1
Samir and Andre will pray Jum'at at Masjid Al-Huda together next Friday. Since the site is further than the previous masjid, they will ride Samir's motorbike instead of Andre's. There are two routes from Samir's house to Andre's house and four routes from Andre's house to Masjid Al-Huda.

a. What good deeds did Samir perform in order to represent a good Muslim? Keep in mind QS Al-Maidah:2

وَتَعَاوَنُوا عَلَى الْبِرِّ وَالْبَرَاءَةِ وَلَا تَعَاوَنُوا عَلَى الْإِثْمِ وَالْعَدْوَانِ إِنَّ اللَّهَ بِشَـدِيدِ الْعِقَابِ ﴿٢﴾

“...And cooperate in righteousness and piety, but do not cooperate in sin and aggression. And fear Allah; indeed, Allah is severe in penalty”.

b. Create your own map to choose the best route! (Hint: use roads as connecting lines.)

c. How can you determine the number of routes Samir may take from his residence to Andre's house and then to Masjid An-Nur? Explain your solution!

2) Sub-activity 2

a. Please number the routes based on Sub-Activity 1(b).

b. How many different Samir travel routes do you have?

c. Explain your conclusion from the road number selections in (a) and the total travel routes in (b)!

In identifying the Islamic value within the given context (1 a), most of students could mention one good deed that could be practiced in their daily life. Figure 1 shows an example of student work in determining Islamic ideals of helping people in need.

![Figure 1](image1)

Figure 1. An example of a student's response in identifying one excellent Islamic virtue

Although the concepts of multiplication rules and how to employ them in identifying possible routes had not yet been explicitly presented to the students, the majority of them were able to rediscover them using their own ways. In other words, by examining the given Islamic context (1 b) might prompt the students to consider multiplication rules. Figure 2 appears to show a student's reaction to items 1b and 1c.

![Figure 2](image2)

Figure 2. An example of a student's response when rediscovering the multiplication rule
In Figure 2, the student drew her own map of the possible route from Samir to Masjid An-Nur. The student drew a line to indicate possible routes from Samir’s house (S) to Andre’s house (A) before heading to Masjid An-Nur. The student used numbers and letters to represent and number the various routes. The student mapped out the only route she could take from Samir's house to the masjid. She then enumerated the total routes from the available options, as shown in Figure 3.

![Figure 3](image1.png)

**Figure 3.** An example of a student’s response in determining the total routes

Following group discussion, most students discovered how to determine probable routes by drawing their own map and interpreting it to a simple diagram, as shown in Figures 2 and 4. In fact, there were students who could model and understand the difficulties into numbers that represented possible routes. The number “2” in Figure 4 written at the top right symbolized the number of possible routes from Samir’s house to the mosque through the first road from Andre's house, and so on (See Figure 4 at the right side). Furthermore, the student added up to 2 as many as 4 which were the possible number of routes from Andre's house to the mosque. This would be led the student to the concept of multiplication rules.

![Figure 4](image2.png)

**Figure 4.** An example of a student’s response to the notion of multiplication rule

All learning activities and students’ responses above indicated that the LT developed in this research could support pupils in reconstructing ideas about multiplication rules. The contextual problems could facilitate in students’ reasoning and justification, developing students’ own strategies and collaboration. All of those activities are the embodiment of the key principle of RME (Fauzan et al., 2013; Gravemeijer, 1994; Heuvel-panhuizen & Drijvers, 2014; Zulkardi, 2002). In addition, the contextual problems which included Islamic values could strengthen students’ Islamic profile/value that they might have before (Muslimin et al., 2020; Putri & Aisyah, 2020).

4. Conclusions

The learning trajectory (LT) of multiplication rules based Realistic-Islamic settings could promote students understanding and reasoning about multiplication rules ideas. Four learning series: (1) a map exploration for going to Masjid, (2) finding a best route for Jum’ah prayer, (3) design your map, and (4) be a good Muslim. This approach helps students reassemble their knowledge of multiplication rules.
in order to predict potential outcomes. The Islamic culture may, most significantly, impart positive morals and acts that can be used in daily life.

Author Contributions
Yoga Dwi Windy Kusuma Ningtyas: conceptualization, methodology, validation, analysis, writing – original draft, writing – review & editing. Chusnul Khotimah Galatea: resources, methodology, analysis, writing – review & editing. Naufal Ishartono: validation and supervision. Naufal Husein Adikalan: project administration and data curation.

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Declaration of Competing Interest
No conflict of interest is declared by authors.

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