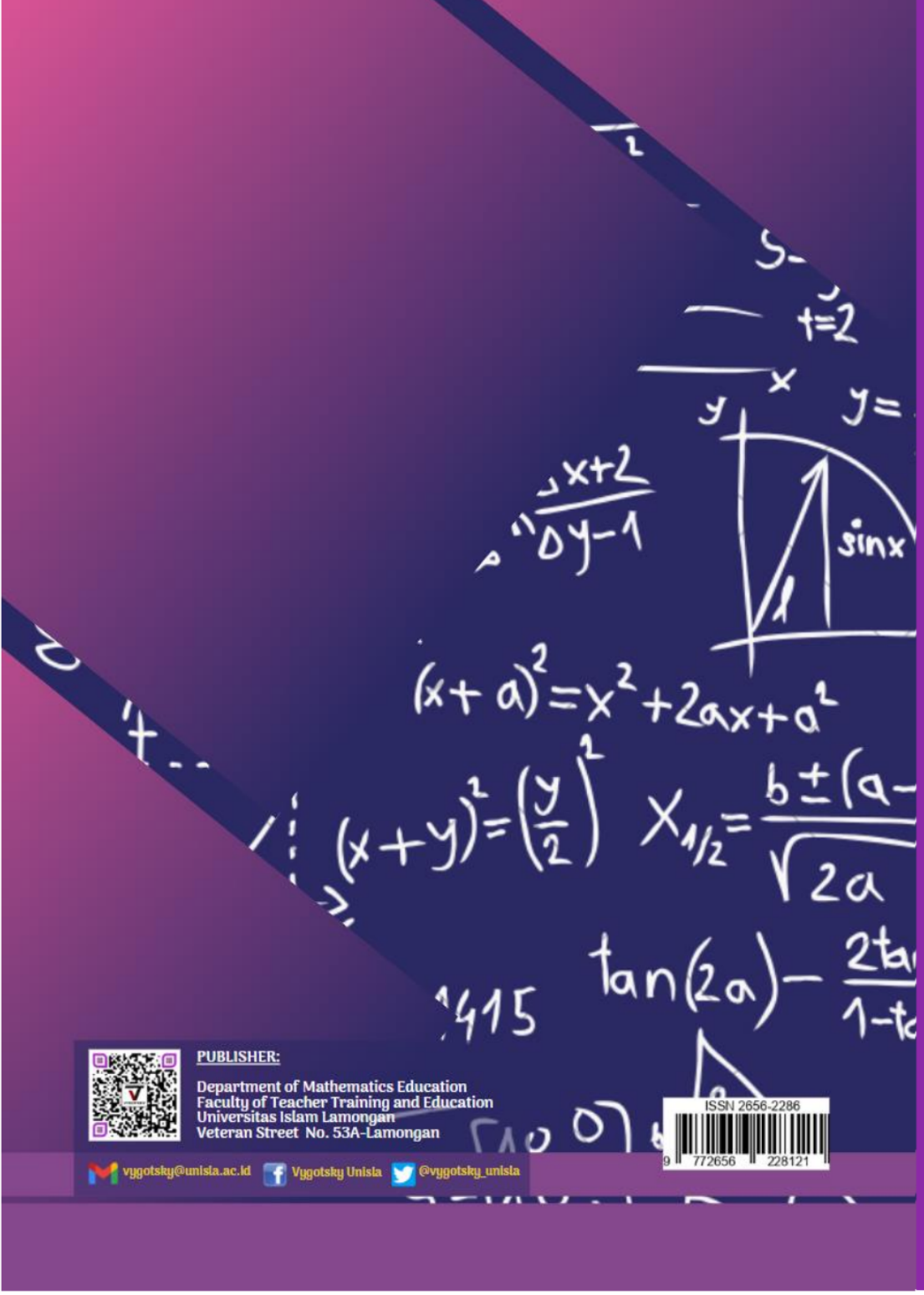


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Table of Contents

Title of Article	Page
An Ethnomathematics Study of Flat-Building Geometry on Traditional Central Javanese Musical Instruments **[*Rifa Aulia Ratih, Rita Pramujiyanti Khotimah*]**	1 - 12
The Effect of the Application of Geogebra Software Android Version on the Mathematical Ability of Politeknik Ilmu Pelayaran Makassar Cadets **[***Fitriani Fitriani, Welem Ada', Zulkaidah Nur Ahzan***]**	13 - 20
Development of a SEIR Model with Two Optimal Controls for Infectious Disease Control: Computational Simulation Using MATLAB [*Sitti Rosnafi'an Sumardi, Muhammad Hisyam*]	21 - 30
Assessing Contextual Mathematical Literacy of Nautical Cadets Using PISA Proficiency Levels: Evidence from Maritime Vocational Education [*Novitasari Novitasari, Eka Nurmala Sari Agustina, Atika Tirzha Sabrina Anggraeni*]	31- 40
Enhancing Students' Mathematical ProblemSolving Skills through Problem-Based Learning in Systems of Linear Equations in Two Variables at Junior High School Level *[Mia Indriyati, Iesyah Rodliyah]*	41 - 50
Estimation of Dengue Fever Transmission Model in West Java Using the Ensemble Kalman Filter Method **[*Addinda Nur Ameliyah, Didik Khusnul Arif*]**	51 - 64
The Effect of GeoGebra-Based Instruction on Students' Conceptual Understanding of Exponential Functions **[***Amalia Rasikhah Earliand, Alfisyahra Alfisyahra, Muh Hasbi, Dasa Ismaimuzaa***]**	65 - 76
Optimization of Sandblasting Project Scheduling at PT. Prima Saka Engineering Using the Hungarian Algorithm **[***Jihan Tria Marena, Nia Nurfitria***]**	77 - 88

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An Ethnomathematics Study of Flat-Building Geometry on Traditional Central Javanese Musical Instruments

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ABSTRACT

The purpose of this study is to explore the mathematical elements contained in Central Javanese gamelan musical instruments by applying qualitative research methods and ethnographic approaches. Data were collected through observation, interviews, and documentation, analyzed using Miles and Huberman's model, with validity ensured through triangulation. The findings indicate that Central Javanese gamelan instruments embody various two-dimensional geometric shapes, including squares, rectangles, circles, and isosceles trapezoids, represented across the kethuk kempyang, bonang, peking, gender, saron, kempul, and gambang. The results of ethnomathematical exploration on traditional Central Javanese musical instruments can be applied to mathematics learning, particularly flat or two-dimensional geometry, by providing visual representations of geometric shapes found in Central Javanese gamelan instruments.

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1. Introduction

Mathematics education plays a role in training the ability to think logically, analytically, and critically, so that the next generation of children is better prepared to face obstacles and take advantage of opportunities that exist in the future (Siregar et al., 2024). It can be explained that mathematics is a science derived from human experiences that aims to prepare the younger generation to be able to think

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logically, analytically, and critically. Geometry material is a mathematical subject in education. Students find this subject difficult to learn. This difficulty creates obstacles for students and prospective teachers and math teachers. Students require improved understanding of geometry concepts.

Geometric concepts such as circles, rectangles, squares, and trapezoids can be identified through the forms and arrangements of traditional musical instruments. A circle consists of points equidistant from a fixed center. Its elements include the center, radius, diameter, arc, chord, sector, and segment. According to Nur'aeni & Muharram (2016), a rectangle has four right angles, two pairs of equal parallel sides, and two lines of symmetry.

According to the research by Zaky & Khotimah (2024), a square is a quadrilateral with four sides of equal length and four equal angles of 90° because it forms right angles. A square also has four axes of symmetry and four axes of rotational symmetry. Quoted from Suharjana (2008), an isosceles trapezoid is a trapezoid with two parallel sides and two legs or vertical sides of equal length, and none of its angles are right angles. These geometric forms provide a mathematical perspective for understanding how traditional instruments are constructed and arranged.

Integrating Gamelan-based ethnomathematics media connects abstract geometric concepts with tangible cultural artifacts, enabling students to observe shapes directly and develop deeper mathematical reasoning through engaging contextual learning experiences effectively. In learning two-dimensional geometry, difficulties are still found, including difficulties in understanding basic concepts, challenges in solving or finding a way out of the problem completely, difficulties in applying material from imaginative forms to real objects, obstacles in applying stages to perform mathematical procedures, and incomprehension in solving problems contained in the problem (Ali et al., 2023).

This situation can arise because students do not understand the parts and characteristics of two-dimensional shapes. Students are unable to identify the necessary parts which may be caused by students' inability to visualize the given problem (Sulistiowati, 2022). Therefore, there is a need for activity-based mathematics learning that connects mathematical concepts with students' cultural environment. One approach is ethnomathematics, which integrates mathematics with cultural contexts to make learning more meaningful (Auliya, 2018). Culturally rooted learning is a way of teaching that prioritizes student activities with the many diverse cultural backgrounds they have in society, this method can be incorporated into the process of teaching certain lessons and assessing learning outcomes, can be assessed with a variety of assessments (Sardiyo & Pannen, 2005).

Ethnomathematics is the study of cultural communities, historical heritage in indigenous peoples, and various other things related to mathematics and the process of learning mathematics (Richardo et al., 2020). In addition, ethnomathematics can also be described as a method that is rarely used in mathematical activities by certain community entities and cultural activists

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(Machaba & Dhlamini, 2021). Through play activities, students can apply math learning concepts such as counting the number of tiles in a room, traditional games (engklek, clogs, etc.), in order to facilitate the concept of area measurement (Manurung, 2018). There are many variations of math learning media, such as textbooks, digital materials, math manipulatives, educational software, and various other tools (Nainggolan, 2023).

Based on information from Hadi et al. (2021), there are 1,239 traditions that have been recognized as tangible and intangible cultural heritage from Indonesia. Central Java Province is one that has cultural diversity, one of which is regional musical instruments. There are various traditional musical instruments of Central Java including bonang, saron, gong, gender, demung, kenong, etc. Culturally rooted learning prioritizes student activities based on diverse societal cultural backgrounds (Sardiyo & Pannen, 2005). Educators incorporate this method into specific lesson processes and learning outcome assessments. Previous studies extensively explore general ethnomathematics applications in mathematics education. Existing literature reveals a critical gap regarding the targeted integration of Gamelan instruments into geometry instruction and assessment. This specific study presents the results of an exploration into the spatial mathematical elements of Central Javanese gamelan instruments.

Previous research results mentioned that the traditional musical instrument Kenong can help teachers in applying ethnomathematics in the teaching and learning process in the classroom (Sari et al., 2022). Research by Fajarianty et al. (2022), also showed the results that the musical instrument drums in Javanese gamelan can convey information about mathematical concepts on the curved side of a truncated cone. In research from Mu'asaroh & Noor (2021), it was concluded that the exploration of tambourine musical instruments could be used as a learning resource in learning geometry at school. In addition, based on research by Prastowo et al. (2022), it is stated that the types of traditional gong musical instruments such as kempul, suwukan, and gong can represent mathematical concepts, namely the concept of relations, circle equations, three dimensions in the form of tubes and hemispheres and integral calculus. Based on the results of the research that has been carried out, this research aims to explore the elements of two-dimensional geometry contained in traditional gamelan musical instruments in Central Java.

2. Method

The qualitative method with an ethnographic approach was chosen as the method of this research. The ethnographic research method in this qualitative research involves researchers investigating the culture of a group that can be understood through observation in its natural environment and interviews (Ramdhan, 2021). The data used came from primary sources obtained from observations, interviews and documentation at the studio in Sukoharjo Regency, with five main narrators, namely the puppeteer, trainer, two pengrawit and studio owner. Data validity

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used triangulation techniques and sources. The data from the current investigation were examined utilizing the Miles and Huberman methodology. Data processing procedures are carried out in a way that includes data reduction, data presentation, and conclusion making.

3. Results and Discussion

3.1 Result

Interviews with several informants (dalang, karawitan teachers, pengrawit, and studio owners) provided some information about traditional musical instruments in the Central Java region. Traditional musical instruments originate from a particular region or culture passed down from ancestors or predecessors. Visually, the form has not changed. Traditional musical instruments in the Central Java region are gamelan consisting of saron, demung, slenthem, drum, gong, kenong, xylophone, gender, kempul, bonang, kethuk kempyang. The technique in playing the instrument is by dipithet (held by hand) and hit. If the pithetannya is not good, the resulting sound will echo.

The following information is the result of interviews with a number of informants (dalang, karawitan teachers, pengrawit, and studio owners) about the use of traditional musical instruments in the Central Java region. Traditional musical instruments in Central Java have several uses, one of which is as a traditional or religious ceremony, for example when there is a wedding, then Tayub, and Wayangan. Traditional musical instruments in Central Java, especially in Solo, can be used as a medium for proselytizing to convert to Islam. There are Guntur Madu and Guntur Sari gamelan in the Palace, which were played by Sunan Kalijaga to invite people to come to the mosque. People who want to come and see the gamelan are given conditions by Sunan Kalijaga to read the creed, so that in the past one of the uses of this traditional musical instrument was for religious propagation.

The interview findings reveal several geometric elements in traditional musical instruments from Central Java, particularly circles, semicircles, rectangles, cylinders, and trapezoids. The circular shape is evident in the outer part of the gong and in the diameter of the drum skin. The central part of the gong forms a convex shape resembling a hemisphere, which is also a geometric representation of a curved surface. The rectangular shape appears in the individual wilahan of the saron, demung, peking, and gambang. When several rectangular wilahan are arranged sequentially according to their respective sizes, their overall arrangement forms a trapezoidal pattern. These geometric forms are not merely aesthetic but are closely related to the mathematical calculations used to determine the appropriate dimensions and proportions of the instruments so that the resulting tones are accurate and harmonious.

Furthermore, data regarding geometric elements in traditional Central Javanese musical instruments were obtained from the results of interviews,

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observations, and documentation conducted, with the following results:

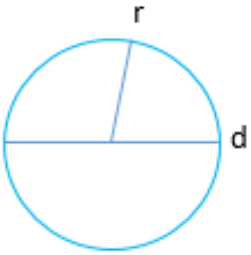
1. Kempul

   In the traditional kempul musical instrument, there is a mathematical element in the form of a circular two-dimensional shape, as shown in Figure 1.



Figure 1. Kempul

Figure 1 can be visualized in the following form:



Circle

This illustration is circular because the kempul has a radius and diameter.

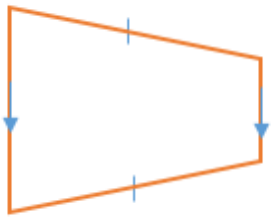
2. Peking

   In traditional peking musical instruments, mathematical elements are found in the form of rectangular two-dimensional shapes and isosceles trapezoids as shown in Figure 2.

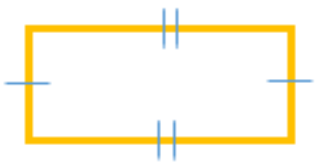


Figure 2. Peking

Figure 2 can be illustrated in the following form:



(1) Iscosceles



(2) Rectangle

The combined peking wilahan illustrated in figure (1) is in the shape of an isosceles trapezoid, as it has parallel sides on the base side, and two equal leg sides. Figure (2) illustrates a rectangular Peking wilahan because it has two opposite sides that are equal in length.

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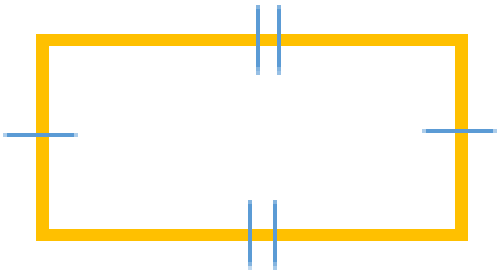
3. Gender

   In traditional gender musical instruments, mathematical elements are found in the form of rectangular two-dimensional shapes as shown in figure 3.



Figure 3. Gender

Figure 3 can be illustrated as follows:



(1) Rectangle

As illustrated in figure (1), a gender wilahan and a row of gender wilahan have a rectangular shape because they have two sides that face each other of equal length.

4. Kethuk Kempyang

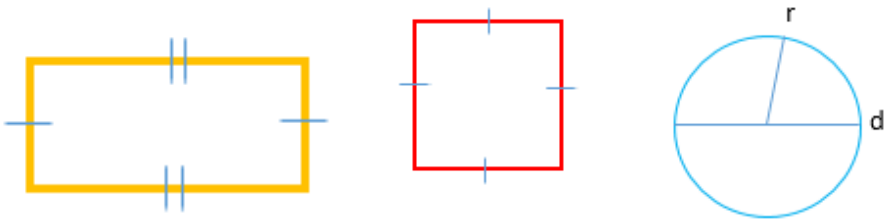
   In the traditional kethuk kempyang musical instrument, mathematical elements are found in the form of square, rectangle, and circle two-dimensional shapes as shown in figure 4.



Figure 4. Kethuk kempyang

Figure 4 can be illustrated as follows:

E-ISSN: 2656-5846
P-ISSN: 2656-2286



(1) Rectangle (2) Square (3) Circle

The box on the kethuk kempyang illustrated in figure (1) is rectangular because it has two sides facing each other of equal length. Figure (2), illustrates the bulkhead on the kethuk kempyang is square because it has 4 sides that are the same length. While figure (3), the illustration of kethuk kempyang is circular because it has a radius and diameter.

5. Gambang

The traditional gambang musical instrument found mathematical elements in the form of rectangular two-dimensional shapes and isosceles trapezoids as shown in figure 5.

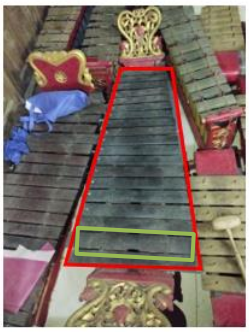
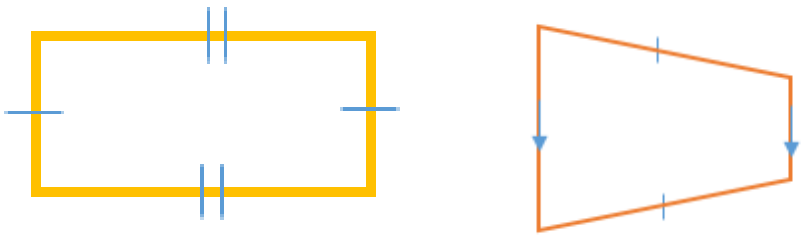


Figure 5. Gambang

Figure 5 can be illustrated as follows:



(1) Rectangle (2) Isosceles trapezoid

The wilahan on the gambang illustrated in figure (1) is rectangular because it has two sides that are opposite each other and of equal length. Meanwhile, figure (2) shows an illustration of a combined wilahan gambang in the shape of an isosceles trapezoid, because it has parallel sides at the base and two sides of equal length.

6. Bonang

In the traditional musical instrument bonang, mathematical elements in the form of two-dimensional shapes such as squares, rectangles, and circles can

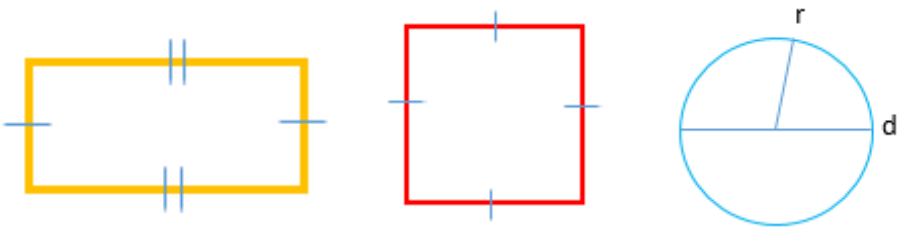
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be found, as shown in figure 6.



Figure 6. Bonang

Figure 6 can be illustrated as follows:



(1) Rectangle (2) Square (3) Cicle

The large bonang box illustrated in figure (1) is rectangular because it has two sides that are the same length. Figure (2) shows a bonang partition that is square because it has four sides that are the same length. Meanwhile, figure (3) shows a bonang that is circular because it has a radius and diameter.

7. Saron

In the traditional musical instrument saron, mathematical elements are found in the form of rectangles and isosceles trapezoids, as shown in figure 7.

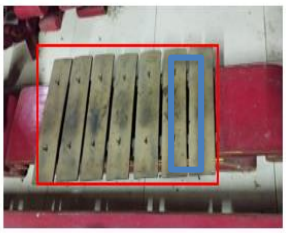


Figure 7. Saron

Figure 7 can be illustrated as follows:



(1) Rectangle (2) Isosceles trapezoid

The saron wilahan illustrated in figure (1) is rectangular because it has two sides that are opposite each other and of equal length. Figure (2) shows a combined illustration of a saron wilahan in the shape of an isosceles trapezoid, because it has parallel sides at the base and two sides of equal

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length.

3.2 Discussion

Based on this research, it can be understood that traditional musical instruments originate from a region or culture that is passed down from generation to generation by ancestor. Traditional musical instruments originate from the term tradition, which is related to customs passed down from ancestors to future generations, according to research conducted by (Salsabil, 2022).

Statements obtained from sources can be seen froms their form; there are mathematical elements in traditional Central Javanese musical instruments, such as the gong. If mathematical elements were not used, the form would be chaotic and irregular, resulting in a sound that would not resonate. In line with the research conducted by Nurzaytun & Susilowati (2025), it is stated that every musical instrument not only functions as an instrument but also reflects the geometric principles embedded in Nusantara culture. The physical forms of instruments such as gongs, saron, bonang, flutes, kendang, kempul, and demung, and their mallets demonstrate the concept of diversity in spatial and two-dimensional shapes.

The findings of this study indicate that the geometric shapes found include squares on the kethuk kempyang and bonang musical instruments, rectangles on the peking, gender, kethuk kempyang, bonang, and saron musical instruments, circles on the kempul, kethuk kempyang, and bonang musical instruments, and isosceles trapezoids on the peking, gambang, and saron. This is in alignment with the perspective of Valda et al. (2022), who state that two-dimensional shapes consist of several types, such as circles, isosceles trapezoids, squares, and rectangles.

Geometry is a branch of mathematics that focuses on shapes, sizes, and spatial relationships. Plane geometry specifically deals with two-dimensional shapes characterized by length and width, such as squares, rectangles, circles, and trapezoid (Zhu et al., 2021). The findings of this ethnomathematical exploration show that these geometric concepts are represented in the physical forms and arrangements of several traditional Central Javanese musical instruments. Squares and rectangles can be observed in several instruments, while circular forms are found in the kempul, kethuk kempyang, and bonang, and isosceles trapezoidal forms are identified in the peking, gambang, and saron.

These findings are relevant to students' difficulties in learning geometry, particularly in visualizing geometric concepts and relating them to real-world objects. The geometric forms identified in gamelan provide concrete visual examples that help students recognize, describe, compare, and analyze shapes. Thus, gamelan can contextualize abstract geometric concepts through tangible cultural objects and support meaningful geometry learning. This finding is consistent with (Sari et al., 2022), who reported that traditional Kenong musical instruments can be utilized by teachers as a context for implementing

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ethnomathematics in mathematics learning. Their findings support the view that traditional musical instruments can function not only as cultural artifacts but also as concrete learning resources through which students can explore mathematical concepts.

The use of gamelan as a learning context also strengthens the application of ethnomathematics in mathematics education. By integrating traditional musical instruments into geometry learning, teachers can present learning activities that connect mathematical concepts with students' cultural surroundings. This approach can make geometry more contextual and meaningful while helping students develop their ability to visualize and apply geometric concepts to real-world objects. Furthermore, Prastowo et al. (2022) demonstrated that traditional gong instruments, including kempul, suwukan, and gong, can represent various mathematical concepts, such as relations, circle equations, three-dimensional shapes including cylinders and hemispheres, and integral calculus. These findings reinforce the present study by demonstrating that traditional musical instruments embody diverse mathematical concepts beyond plane geometry. The identified geometric forms can support culturally relevant mathematics learning materials, enabling students to progress from concrete visual experiences toward abstract concepts while promoting contextual, accessible, and culturally meaningful geometry learning.

4. Conclusions

Central Javanese traditional musical instruments, particularly gamelan, are not only used in traditional ceremonies and cultural activities but also contain mathematical elements that can be identified in their physical forms. The findings of this study show that several gamelan instruments represent two-dimensional geometric shapes, including circles found in the kempul, kethuk kempyang, and bonang; squares in the kethuk kempyang and bonang; rectangles in the peking, gender, kethuk kempyang, bonang, and saron; and isosceles trapezoids in the peking, gambang, and saron. These findings demonstrate that geometric concepts can be observed in cultural objects that are familiar within students' everyday environments.

The identification of these geometric elements has pedagogical implications for mathematics education. Integrating gamelan into geometry learning provides concrete visual contexts that help students recognize, visualize, describe, compare, and relate abstract concepts to real-world objects. This approach supports conceptual understanding, spatial and visualization skills, cultural literacy, and meaningful geometry learning through local cultural heritage.

Author Contributions

Rifa Aulia Ratih was responsible for designing the research, conducting fieldwork including observations and interviews, collecting and analyzing the data, and drafting the manuscript. Rita Pramujiyanti Khotimah provided research

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supervision, contributed to the refinement of the research framework and analysis.

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Declaration of Competing Interest
This study belongs to the author. There is no conflict of interest for this study.

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The Effect of the Application of Geogebra Software Android Version on the Mathematical Ability of *Politeknik Ilmu Pelayaran Makassar* Cadets

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ABSTRACT

Mathematics is used in various aspects of life, including the maritime and marine world. Mathematical concepts are widely used to solve problems in this field. Understanding abstract mathematical concepts requires a tool to bridge these concepts into concrete forms. The aim of this study is to determine the effect of using the Android version of GeoGebra software in improving the mathematical abilities of cadets. GeoGebra is a comprehensive and easy-to-use mathematical software. A study conducted on cadets at *Politeknik Ilmu Pelayaran Makassar* found that the use of the GeoGebra application had a positive impact on their understanding of mathematical material. This was demonstrated by inferential statistics tests and supported by the cadets' responses to the application's use in learning.

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1. Introduction

Mathematics is a fundamental science that is essential and can be applied in science, technology, agriculture, economics, education, and other fields, including maritime and marine affairs. Some instances of applications or problems of mathematics in the maritime field include calculating angles to determine the location of objects, the Cartesian system to identify the location of an object, flat and spatial geometry to determine the area or calculate the volume of hydraulic

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systems, navigation, and various other technical calculations related to ship operations, among others (Agus et al., 2024). However, the abstract nature of mathematical concepts often hinders students' understanding of the concepts themselves (Ninung et al., 2025)(Suciati et al., 2022). Abstract mathematics must be interpreted concretely to facilitate comprehension (Edo & Renoat, 2024). In this context, a tool is needed to help students understand these concepts effectively.

One of the educational providers in the maritime and marine sector is the *Politeknik Ilmu Pelayaran Makassar*, specifically its nautical study program. This program produces graduates with competencies as shipping officers. Graduates of this program can also work in other sectors such as mining, the defense industry, and other maritime sectors. One of the subjects in this program is applied mathematics. This subject is offered because its application can be used for calculating ship position, navigation, and various other technical calculations related to ship operation.

In technology-based mathematics learning and in the digital era, a tool is needed to help male and female cadets at the *Politeknik Ilmu Pelayaran Makassar* understand mathematical concepts and strengthen their mathematical abilities. In the field of mathematics, there are various mathematical software that can be used, including Maple, Matlab, Geogebra, and so on. One simple mathematical application that covers many concepts and can be applied in solving mathematics is Geogebra. This software is lightweight software that can be applied via PC or Android cellphone. In addition, Geogebra software also uses a small storage capacity. By using this ligh and flexible software in terms of the devices used, the researcher hopes that the use of this device will be an alternative that can improve the mastery of concepts and mathematical abilities of cadets at the *Politeknik Ilmu Pelayaran Makassar*. This study aims to determine the effectiveness of using software in improving mathematical abilities of cadets at the *Politeknik Ilmu Pelayaran Makassar*.

2. Method

This research is quantitative pre-experimental design research, namely one-grup pretest-posttest design (O1 X O2), with the treatment given is the application of GeoGebra software media android version. The sampling technique used was a cluster sample with a total sample of 159 cadets. Learning was conducted in a blended manner over 10 sessions, with one session for the pre-test, eight sessions for blended learning, and one session for the post-test. Before the treatment was given, the sample was given a pre-test to determine mathematical abilities and after the treatment was given a final test. The research was conducted at the *Politeknik Ilmu Pelayaran Makassar* in January 2024. The population in this study were all male and female cadets at the *Politeknik Ilmu Pelayaran Makassar* who were currently programming mathematics courses.

The data obtained in this study were mathematics ability in the form of pretest and posttest results, as well as student response questionnaires. The study began with observations at the *Politeknik Ilmu Pelayaran Makassar* campus, preparing research instruments, and coordinating with the head of the Nautical study program. Next, a pretest was conducted, followed by the provision of mathematical materials and concepts through online learning and the assistance of GeoGebra software in the experimental class for approximately four meetings. A posttest was then administered. After data collection, descriptive and inferential

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data analysis were conducted. The inferential analysis began with a normality test using software assistance for the pretest and posttest data. After that, a homogeneity test was conducted, followed by a comparative test using the student *t*-test.

3. Results and Discussion

Before administering treatment to the *Politeknik Ilmu Pelayaran Makassar* cadets, they were given an initial test to determine their initial abilities, particularly in geometry. This initial test consisted of ten multiple-choice questions, each weighted at ten points. After the treatment, a final test was administered with different questions, but with the same difficulty level and number of questions. Overall, the average, median and range of scores for the initial and final tests are presented in the following graph.

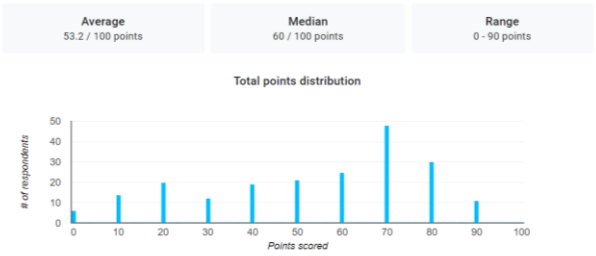


Figure 1. Initial Test Score Graph

Figure 1 shows that the lowest value obtained by respondents is still 0 and the highest value is 90. The middle and average values are not that far apart, indicating that the initial test value data is centered on the average and median value range.

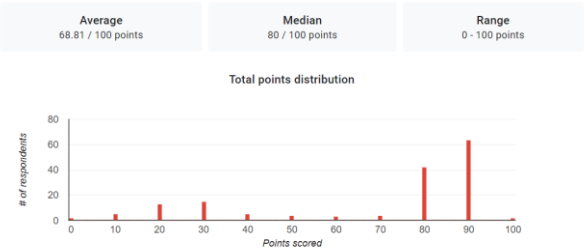
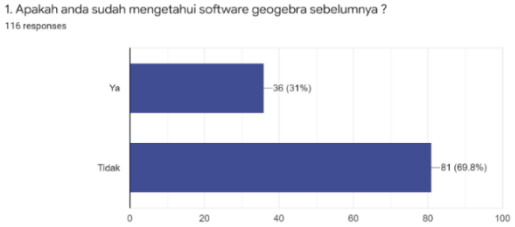


Figure 2. Final Test Score Graph

Figure 2 illustrates that the range of scores obtained by respondents in the final test is 0 to 100. The respondents' scores are centered around 80-90. This indicates an increase in scores on the final test. It can be seen from Figures 1 and 2 that the average increase in scores from the initial test to the final test is 15.61%.



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Figure 3. Response Regarding Knowledge of GeoGebra Software

Figure 3 is the questionnaire responses from respondents, namely male and female cadets of the Nautical Study Program. It was found that 31% of respondents were previously familiar with the GeoGebra application, while 69% were not. The relevant question of Figure 3 was converted to "Do you know about GeoGebra before?".

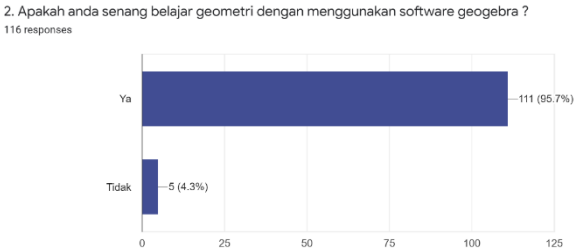


Figure 4. Responses Regarding the Pleasure of Using Geogebra

The question of Figure 4 is literally translated as "Do you derive pleausure from learning Geometry through the use of GeoGebra software?". As many as 95.7% of respondents stated that they enjoyed learning mathematics, especially geometry, using this software. One of the most common reasons respondents cited was that GeoGebra is very easy to use because it is a lightweight application. They also found it very easy to use, as it's available as a PC and Android app.

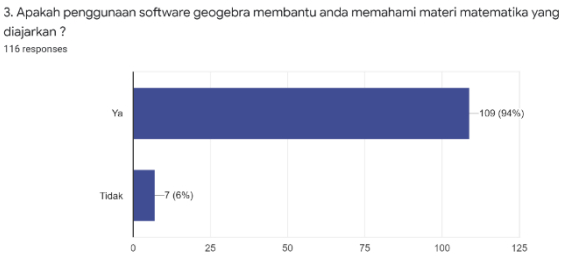
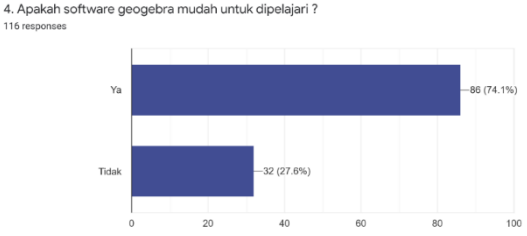


Figure 5. Responses Regarding GeoGebra's Ease of Use in Understanding the Material

The question of Figure 5 is directly translated as "Does GeoGebra Software help you to understand the mathematical material being taught?". Respondents commented that the app is very helpful in solving math problems with its graphical display. The graphics might be two- or three-dimensional. Up to 94% of respondents indicated that utilizing GeoGebra makes learning math easier.



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Figure 6. Responses about GeoGebra are Easy to learn

Figure 6 contains the equation translated to "Does GeoGebra Software easy to learn?". The software was deemed user-friendly by 74.1% of respondents in terms of its simplicity of learning. To strengthen the descriptive data analysis, an inferential analysis was conducted to strengthen the assumptions of the research hypothesis. Before conducting statistical tests to determine the effect of a treatment, a prerequisite test, namely the normality test, is required. Once these two tests have been conducted and fulfilled, the *t*-test can be continued to determine the effect of a treatment. Statistical testing will be conducted using SPSS software to reduce the error rate and inaccuracy of manual calculations. The following are the results of the normality test for the initial and final test data.

Table 1. Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Initial Test	.138	159	.307	.933	159	.780

a. Lilliefors Significance Correction

Based on the SPSS program, the significance value was 0.307 > 0.05, indicating that the initial test data met the normality requirements. The results of the normality test for the initial test were similar. Similarly, the results of the normality test for the final test were 0.423 > 0.05, indicating that the final test data also met the normality requirements. This can be seen in the SPSS results table below.

Table 2. Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Final Test	.306	159	.423	.744	159	.810

a. Lilliefors Significance Correction

Moreover, a homogeneity test was no longer necessary because the samples used came from a single sample class. The following explains the *t*-test results obtained using SPSS.

3.1 Paired Sample t-Test

This study took one sample class and received treatment. Before the treatment, there was a pre-test on mathematics ability, specifically geometry, and after the treatment, a post-test on mathematics ability, specifically geometry, was given with different questions but at the same level of difficulty. Thus, to test whether the treatment had an effect, a paired-samples t-test was used.

Table 3. Paired Samples Statistics

		Mean	**N**	**Std. Deviation**	**Std. Error Mean**
Pair 1	Initial Test	44.8428	159	26.76327	2.12247
	Final Test	74.9686	159	20.89377	2.65698

The average of the final test was higher than the average of the initial test, indicating a difference in the averages with a sample of 159 respondents. Furthermore, to determine whether the difference is truly real (significant) from the increase in the average of the first and final tests, see Table 4.

E-ISSN: 2656-5846
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Table 4. Paired Samples Correlations

		N	Correlations	Sig.
Pair 1	Initial and Final Test	159	.599	.412

Table 4 shows the results of a correlation test, or relationship, between the two sets of data: the initial and final tests. Based on the significance value of 0.412 > probability of 0.05, it can be concluded that there is no relationship between the two variables in the initial and final tests.

Table 5. Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper			
Pair 1	Initial Test - Final Test	30.12579	30.50187	2.41896	34.90345	25.34813	12.454	158	.007

The output table above is the most important output because in this section, it can be seen whether there is an effect of the treatment or not. Based on the table, the Sig. (2-tailed) value is 0.007 < 0.05, which indicates that there is a difference in the average results of the initial test and the final test, so it can be said that there is an effect of the treatment, namely learning using the GeoGebra application.

Based on the descriptive results and the results of the inferential analysis, it was found that there was an effect of treatment in the form of teaching using media in the form of the GeoGebra application on the mathematical abilities of male and female cadets of the Nautical Study Program *Politeknik Ilmu Pelayaran Makassar*. This was also supported by qualitative response data from the cadets, which stated that the GeoGebra application was very helpful in understanding and learning mathematics, especially geometry.

According to (Zetriuslita et al., 2020), learning mathematics using media can generally help students learn independently. The availability of GeoGebra software in the Android version can also help independent learning, especially in the topic of geometric mathematics for male and female cadets. The same thing was also stated by (Saha et al., 2010)(Negara et al., 2022), that is to say the importance of using media in mathematics learning that can help students understand mathematical concepts contextually in relation to geometric interpretation. What is considered difficult in learning mathematics is the abstract nature of mathematical material and concepts, so that mathematics learners are required to be able to understand through a more contextual approach (Sari et al., 2024)(Putra et al., 2020). If the concepts and materials are understood through a contextual approach, it is hoped that they will be more meaningful, resulting in the material and concepts being able to remain in the long-term memory of the learners. Research conducted by (Romaito et al., 2021)(Diva et al., 2023) found that the use of media in learning can deepen mastery of mathematical concepts.

Qualitative data obtained from the questionnaire responses of male and female cadets showed a positive response to the use of the Geogebra application

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in mathematics learning. A similar thing was found in research conducted by (Nzaramyimana, 2021) where students were happier and more active in the learning process when using Geogebra media compared to learning without using Geogebra media. In addition, research conducted by (Praveen & Leong, 2013)(Arbain & Shukor, 2015) said that learning independence was formed when students were given Geogebra media in learning abstract mathematical concepts such as in geometry topics. A similar thing was found in this study that the use of GeoGebra software Android version, can help improve the mathematical abilities of cadets at the *Politeknik Ilmu Pelayaran Makassar*.

4. Conclusions

Based on the research results and discussion, it can be concluded that there is an effect of the application of GeoGebra software on the mathematical abilities of cadets in the Nautical Study Program *Politeknik Ilmu Pelayaran Makassar*. This could be analyzed from the increase in scores from the initial test to the final test about 15.61% which is supported by descriptive, and inferential results through hypothesis testing which obtained a significance value of 0.007 <0.05 in the paired one-sample t-test shows a difference in the average pre-test and post-test scores. In addition, learning using this application received a positive response from respondents who attended the lecture as shown through the results of the response questionnaire made in the form of a Google form. Activeness is also reflected in how great the enthusiasm and eagerness of the cadets in participating in learning.

Author Contributions

This paper was completed thanks to the collaboration of a team consisting of three authors. The author team jointly conducted the research until the writing of this article. The first and third authors designed and schemed the research method; the second author contributed more to providing the place and time for the research and implementing the treatment in the form of using GeoGebra software in learning and data collection with the cadets *Politeknik Ilmu Pelayaran Makassar*. During the data analysis, the three authors worked together starting from data input to interpreting the results of the analysis using the SPSS application. Next, the three authors discussed the unique things in the research written in the background and also searched for the latest references to support the results of the research that had been carried out.

Declaration of Competing Interest

The author declares that it has no conflicts of interest.

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Development of a SEIR Model with Two Optimal Controls for Infectious Disease Control: Computational Simulation Using MATLAB

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ABSTRACT

This study develops a Susceptible–Exposed–Infectious–Recovered (SEIR) mathematical model with two time-dependent optimal control strategies educational campaigns and medical treatment to analyze Ebola transmission. The nonlinear system is examined using Pontryagin's Minimum Principle, and the optimality system is solved numerically through the Forward–Backward Sweep Method in MATLAB. Simulation results show that combined controls substantially reduce the peak number of infected individuals, decrease cumulative infection cases, and shorten the epidemic duration compared to the uncontrolled scenario. Early implementation of both interventions further improves epidemic suppression and reduces the overall cost functional value. These results demonstrate the effectiveness of integrated control strategies and highlight mathematical modeling as a decision-support tool in public health planning.

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1. Introduction

Infectious diseases pose one of the greatest threats to global public health, particularly those caused by deadly viruses like Ebola. The Ebola virus first emerged in 1976 and has caused several major outbreaks with high mortality rates, in some cases reaching up to 90% (World Health Organization, 2018)(World Health Organization, 2018). Transmission of the virus occurs primarily through direct contact with the body fluids of infected individuals, making it particularly risky in areas with high population density and inadequate healthcare systems. In

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the context of globalization and high human mobility, the risk of this disease spreading is not limited to Africa but also has the potential to reach other countries, including Indonesia (Sariadji & Kambang, 2022)(K. Sariadji, 2022).

Various scientific approaches have been used to study the spread of infectious diseases, one of which is through mathematical modeling. The SEIR (Susceptible–Exposed–Infected–Recovered) model is a basic and widely used model to describe the dynamics of infectious disease spread (Li et al., 2015)(Li et al., 2015). This model divides the population into four compartments based on infection status and maps the transitions between these compartments over time. It is considered highly representative for modeling diseases like Ebola, which features a notable incubation period and high infection rate. In addition, several studies have shown that the SEIR model can capture local dynamics of transmission when applied to real epidemic scenarios, such as in the case of Ebola in Uganda (Chowell et al., 2004)(Chowell et al., 2004).

To strengthen the model's effectiveness and support real-world decision making, many researchers have adopted optimal control theory**.** The Pontryagin Minimum Principle (PMP) is one of the most widely used mathematical approaches to determine control strategies that can minimize infections and associated costs (Lenhart & Workman, 2007)(Lenhart & Workman, 2007). In epidemiological contexts, optimal control strategies often include public education campaigns aimed at reducing risky behaviors and treatment interventions to accelerate recovery in infected individuals (Funk et al., 2010; Okosun et al., 2013)(Funk et al., 2009)(Okosun et al., 2011). The combination of these two controls has been proven to provide a synergistic effect in suppressing the transmission of infectious diseases (Rachah & Torres, 2015)(Rachah & Torres, 2015).

Solving such complex models analytically is often infeasible due to the nonlinear nature of the equations involved. Hence**,** numerical methods are necessary to find approximate solutions. One of the widely used algorithms for solving optimal control problems is the Forward-Backward Sweep Method (Polak, 1997)(Polak, 1997). For implementation, MATLAB is commonly chosen because of its capability to handle complex differential systems and to produce high-quality visual simulations (Higham & Higham, 2016)(Higham & Higham, 2016).

This study builds upon and extends the SEIR optimal control framework previously introduced by Sumardi (2022)(Sumardi, 2022), which formulated an SEIR model incorporating two time-dependent control variables: public education and medical treatment. While the earlier study primarily focused on analytical derivation and theoretical optimality conditions, it did not provide an in-depth computational investigation of the dynamic behavior of the controlled system.

In response to this gap, the present study repositions the research by emphasizing a computational and quantitative analysis of the dual-control SEIR model. Specifically, this work integrates numerical simulations using MATLAB to systematically examine how the simultaneous implementation of public education and medical treatment influences the temporal evolution of each compartment (Susceptible, Exposed, Infected, and Recovered). Through this approach, the study provides a clearer evaluation of the effectiveness, interaction, and relative impact of the two control strategies in suppressing disease transmission. Therefore, the main contribution of this research lies in bridging the theoretical optimal control formulation with practical computational validation, offering deeper insight into policy-relevant disease mitigation strategies.

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Through this model, the dynamics of the Ebola outbreak can be visualized more realistically and measured in response to different intervention strategies. Therefore, this research contributes not only theoretically to mathematical epidemiology, but also practically to public health planning by providing insight into effective and cost-efficient control interventions that are applicable in real epidemic settings.

In accordance with academic writing conventions, this introduction clearly states the urgency of the study, the theoretical framework used, and the basis for the research. A preliminary literature gap is identified between analytical and simulation-based studies of SEIR optimal control. Therefore, the novelty of this study lies in its implementation of dual optimal controls within a realistic numerical framework, intended to support data-driven health policy.

2. Method

The subject of this study is a mathematical model of Ebola virus spread based on the SEIR (Susceptible, Exposed, Infected, Recovered) approach. This model is a system of nonlinear differential equations that represents the transition of individuals in a population through several health compartments, influenced by two types of optimal interventions or controls: education control for susceptible individuals and treatment control for infected individuals.

Data collection techniques were conducted through literature studies of relevant scientific publications, both national and international, that examine SEIR-based infectious disease modeling and optimal control theory. Parameter values such as infection rate, transition rate between compartments, recovery rate, and mortality rate were adopted from previous journals and references from official health organizations such as the WHO. Parameters were calibrated to match the realistic biological dynamics of Ebola in the simulation.

The initial step in data analysis is to construct a system of differential equations based on the modified SEIR model. The basic model used is as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta_1(1-u_1(t))SI - \beta_2(1-u_1(t))SE - \mu S + \xi R \\
\frac{dE}{dt} &= \beta_1(1-u_1(t))SI + \beta_2(1-u_1(t))SE - (\sigma+\mu)E \\
\frac{dI}{dt} &= \sigma E - (\gamma + \delta + \mu + u_2(t))I \\
\frac{dR}{dt} &= \gamma I - (\mu + \xi)R + u_2(t)I
\end{aligned} \tag{1}$$

Description:

S, E, I, R	: Number of susceptible, exposed, infected, and recovered populations
Λ	: Population entry rate
β_1, β_2	: Transmission rates from I and E
σ	: Transition rate from E to I
γ	: Recovery rate
δ	: Death rate due to disease

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μ : Natural death rate
ξ : Immunity loss rate
$u_1(t) \in [0,1]$: Level of education, reduces the effectiveness of contact (infection).
$u_2(t) \in [0,1]$: Treatment rate, accelerates recovery from I to R.

The equation 1, $u_1(t) \in [0,1]$ represents the time-dependent intensity of public education efforts aimed at reducing effective contact rates. The control variable is defined within the normalized interval [0,1] to reflect proportional implementation, where $u_1(t) = 0$ indicates the absence of educational intervention and $u_1(t) = 1$ represents the maximum feasible level of implementation under practical and resource constraints.

From a modeling perspective, restricting $u_1(t)$ to [0,1] ensures mathematical consistency and boundedness of the control, preventing unrealistic negative efforts or intervention levels exceeding full capacity. Epidemiologically, this normalization allows the control to be interpreted as the fraction or effectiveness of behavioral change induced by education (e.g., mask usage, hygiene practices, reduced physical contact), which proportionally decreases the transmission rate. Thus, the interval [0,1] provides both theoretical tractability in optimal control analysis and a meaningful practical interpretation in public health policy implementation.

Next, Pontryagin's Minimum Principle is applied to obtain optimal control $u_1(t)$ and $u_2(t)$ which minimize the number of exposed and infected individuals while also minimizing the associated intervention costs. The objective functional is defined as:

$$\boldsymbol{J(u_1, u_2) = \int_0^T [AI(t)^2 + B_1 u_1(t)^2 + B_2 u_2(t)^2] dt}$$

Description:
A : weight associated with infected individuals,
B_1 : weight of public education cost,
B_2 : weight of treatment cost.

To enhance clarity, the computational procedure of the proposed optimal control strategy is encapsulated in the subsequent algorithm. Algorithm: Forward–Backward Sweep Method for Dual-Control SEIR Model.

Step 1: Initialization

1. Set parameter values and initial conditions for the SEIR system.
2. Choose final time $T$and discretize the interval $[0, T]$.
3. Initialize control functions $u_1^{(0)}(t)$and $u_2^{(0)}(t)$(e.g., constant initial guesses within $[0, 1]$).
4. Set convergence tolerance ε.

Step 2: Forward Sweep (State System)

5. Solve the state equations (SEIR model with controls) forward in time using the current controls $u_1^{(k)}(t)$, $u_2^{(k)}(t)$.
6. Obtain numerical solutions for $S(t), E(t), I(t), R(t)$.

Step 3: Backward Sweep (Co-state System)

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7. Using the transversality conditions $\lambda_i(T) = 0$, solve the adjoint (co-state) equations backward in time.
8. Obtain $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$.

Step 4: Control Update

9. Update controls using the characterization derived from Pontryagin's Minimum Principle:

$$u_1^{(k+1)}(t) = \min\left\{1, \max\left\{0, \frac{\text{expression involving states and adjoints}}{2B_1}\right\}\right\}$$
$$u_2^{(k+1)}(t) = \min\left\{1, \max\left\{0, \frac{\text{expression involving states and adjoints}}{2B_2}\right\}\right\}$$

10. Ensure the updated controls remain within the admissible interval [0,1].

Step 5: Convergence Check

11. Compute the difference between successive controls:

$$\| u^{(k+1)} - u^{(k)} \| < \varepsilon$$

12. If convergence is satisfied, stop. Otherwise, set $k = k + 1$and repeat Steps 2–5.

Step 6: Evaluation of Effectiveness

13. Compare uncontrolled and controlled scenarios by analyzing:
    a. Peak and total number of infected individuals,
    b. Time to epidemic peak,
    c. Value of the objective functional $J(u_1, u_2)$.

All numerical simulations are implemented in MATLAB (version 2016a) using the Forward–Backward Sweep Method.

3. Results and Discussion

The simulation results clearly illustrate the effectiveness of implementing dual optimal controls—public education and treatment interventions—in minimizing the spread of the Ebola virus within the $SEIR$ model framework. In this study, we developed a SEIR model integrated with two optimal controls, namely u_1(education) dan u_2 (treatment) to control the spread of infectious diseases. Control u_1 is designed to reduce new infections by decreasing the transition rate from the Susceptible (S) to Exposed (E) compartment, representing preventive efforts such as education or contact restrictions.

Meanwhile, control u_2 aims to accelerate recovery by increasing the transition rate from the Infected (I) to Recovered (R) compartment, which can be interpreted as the effectiveness of treatment or clinical intervention. The results of computational simulations using MATLAB are presented in Figures 1, 2, and 3 to illustrate the impact of these optimal controls on population dynamics.

Figure 1 displays the population dynamics of exposed individuals (E) as a function of time (in months) between the no-control (dashed line) and optimal-control (solid line) scenarios. The vertical scale in this graph is in the order of 10^{-3}, indicating that the proportion of the exposed population is very small relative to the total population. In the no-control scenario, the exposed population (E) exhibits progressive exponential growth, reaching a value of approximately 5.5×10^{-3} at the end of the simulation period.

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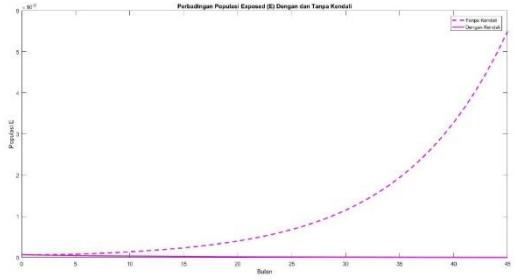


Figure 1. Comparison of Exposed Population (E) with and without Optimal Control

This growth indicates that without intervention, the number of individuals potentially becoming ill will continue to increase. In contrast, in the optimal-control scenario, the exposed population (E) is drastically suppressed to near zero throughout the entire simulation period. This nearly flat curve confirms the high effectiveness of the u_1 control in reducing the transition from S to E, indicating the success of the prevention strategy in suppressing the number of exposed individuals early in the transmission chain.

The nearly flat curve approaching zero indicates that outreach control has been significantly successful in reducing the probability of individuals moving from the vulnerable to the exposed group. This means that public education about the dangers of physical contact, as well as increasing awareness of health protocols, plays a key role in reducing the initial rate of disease spread.

Figure 1 ilustrates the significant reduction in the exposed population under optimal control emphasizes the role of educational campaigns in limiting new exposures. This aligns with findings from Funk et al. (2010)(Funk et al., 2009) and Seck et al. (2022)(Seck et al., 2021), who emphasized that public awareness and behavior change reduce contact rates and transmission pathways. The flatness of the exposure curve in our study suggests a near-elimination of early transmission chains, a result also observed in

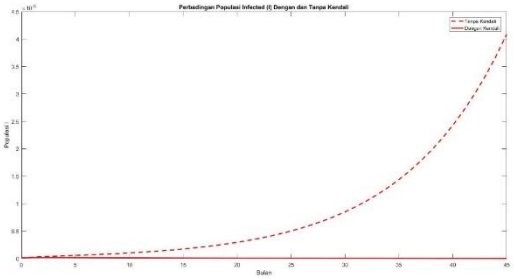


Figure 2. Comparison of Infected Population (I) with and without Optimal Control

Figure 2 shows the population dynamics of infected individuals (I) as a function of time, comparing the uncontrolled (dotted line) and optimally controlled (solid line) conditions. Similar to the exposed population, the vertical scale is in the order of 10^{-3}. In the uncontrolled scenario, the infected population (I) also shows exponential growth, reaching a value of approximately 4×10^{-3} at

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the end of the simulation. This indicates that without intervention, even on a small scale, the number of individuals actively transmitting the disease will continue to increase. However, with the implementation of optimal control, the infected population (I) is successfully suppressed significantly to near zero throughout the entire simulation period. This drastic suppression of I is a direct consequence of the success of the u_1 control in reducing the population E , as well as the contribution of the u_2 control in accelerating the recovery from I to R. These results demonstrate the comprehensive effectiveness of the control strategy in minimizing the burden of infection in the population.

The sharp decline in the infected population in the controlled scenario demonstrates that dual interventions, including education and treatment, can halt the rate of new infections and accelerate the transition to recovery. This demonstrates the synergy between controls, not only breaking the chain of transmission but also reducing the number of individuals experiencing severe symptoms.

Figure 2 shows the infected population drastically declines under optimal control. This supports the synergy between the two controls: public education reduces exposure, while timely treatment accelerates recovery. Similar outcomes were presented by Allali et al. (2024)(Allali et al., 2025), who concluded that the dual-strategy approach significantly lowers infection peaks and shortens epidemic durations. This outcome highlights the value of rapid and concurrent implementation of both control measures.

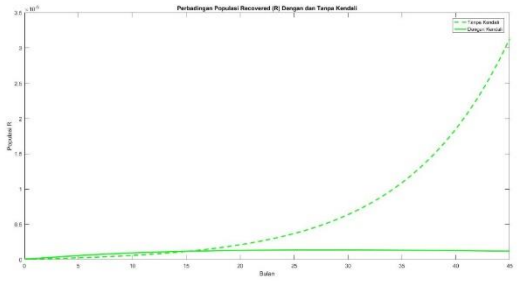


Figure 3: Comparison of Recovered Population (R) with and without Optimal Control

Figure 3 presents the population dynamics of recovered individuals (R) as a function of time, comparing the no-control scenario (dashed line) and the optimal control scenario (solid line). Similar to the E and I, compartments, the vertical scale is also on the order of 10^{-3}. In the no-control scenario, the R population shows continued exponential growth, reaching approximately 3.1×10^{-3} at the end of the simulation, indicating natural recovery. In the optimal control scenario, the recovered population remains low because very few individuals become infected due to early interventions that successfully contain the spread in the early stages. Although the u_2 control is designed to accelerate recovery ($I \rightarrow R$), the low R population in the optimal control scenario can be explained by the remarkable success of the u_1 control.

Since u_1 is highly effective in suppressing the number of exposed (E) and infected (I) individuals to near zero (as shown in Figures 1 and 2), it follows logically that very few individuals can enter the recovered (R). compartment. Therefore, the low R population does not mean that the u_2 control is ineffective,

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but rather is an indication of the overall success of the control system in preventing most individuals from becoming infected and then recovering, thereby reducing the total burden of disease in the population.

Figure 3 shows a lower number of recovered individuals in the controlled scenario. Although this might appear counterintuitive, it actually reflects the model's success in preventing individuals from becoming infected in the first place. This paradox has also been noted by Cacace and Oliviero (2023)(Cacace & Oliviero, 2024), who pointed out that effective early intervention can reduce the need for medical recovery entirely. The model implies that less recovery is not indicative of control failure, but rather a direct result of successful transmission suppression.

The discussion also reveals limitations in the model's structure. It assumes homogeneous mixing and static parameters, which are simplifications that may not reflect real-world epidemic dynamics such as social behavior shifts, viral mutations, or healthcare disparities (Zinihi et al., 2025)(Zinihi et al., 2025). Additionally, the model lacks spatial dynamics, which could affect accuracy in simulating real geographic spread (Hethcote, 2000; Wang & Zhao, 2012)(Hethcote, 2000)(Wang & Zhao, 2008).

Based on these observations, future research should incorporate real-time data integration, spatial heterogeneity, age structures, and adaptive controls. Simulation platforms should also accommodate real outbreak case data to improve the model's predictive accuracy and policy relevance. The implications of this study suggest that dual-control strategies, when initiated early, can be an effective framework for health policy planning, particularly in low-resource settings (Sumardi, 2022; WHO, 2018)(Sumardi, 2022)(World Health Organization, 2018).

4. Conclusions

This study confirms that implementing a dual optimal control strategy educational campaigns and medical treatment within the SEIR model framework can significantly reduce the number of exposed and infected individuals during the outbreak of infectious diseases such as Ebola. The simulation demonstrates that combining these controls from the early stage of an epidemic effectively suppresses transmission, preventing the disease from reaching wider segments of the population.

From a mathematical and epidemiological perspective, the use of Pontryagin's Minimum Principle and the Forward-Backward Sweep Method provides a reliable approach to identifying cost-effective intervention strategies. The findings support earlier research but further emphasize the importance of timing and synergy in public health responses.

Nevertheless, this research is not without limitations. The model assumes homogeneous mixing and constant parameters over time, which may oversimplify real-world complexities such as social behavior dynamics, mutation of the virus, or environmental changes. Additionally, the reliance on hypothetical parameter values without calibration using real outbreak data may limit the accuracy of the projections for policy application.

Future studies should consider integrating real epidemiological data, spatial dimensions, age structures, or stochastic effects to enhance model realism. The inclusion of adaptive control strategies that respond dynamically to data updates could also be explored. Furthermore, interdisciplinary collaboration with public

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health experts and policymakers is crucial to transform theoretical insights into practical action.

The implication of this research lies in its contribution to epidemic preparedness. Mathematical models, when paired with optimization techniques, can serve as decision-support tools for designing responsive, cost-efficient health interventions, particularly in regions with limited healthcare resources.

Author Contributions

The first author contributed to the construction of the model, theoritical analysis, and interpretation of the results. The second author contributed to the numerical analysis and simulation. The first author as corresponding author revised the article based on review from reviewer and journal editor. All authors provided critical feedback and helped shape the research, analysis and manuscript**.**

Declaration of Competing Interest

No potential conflict of interest was reported by the author(s).

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Assessing Contextual Mathematical Literacy of Nautical Cadets Using PISA Proficiency Levels: Evidence from Maritime Vocational Education

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ABSTRACT

Mathematical literacy is a crucial competence in maritime education, supporting navigation accuracy, operational efficiency, and safety-related decision-making. However, empirical studies assessing mathematical literacy within authentic maritime vocational contexts remain limited. This study investigates the contextual mathematical literacy of nautical cadets using the PISA proficiency framework embedded in authentic maritime work-related scenarios. A quantitative descriptive design was employed with 23 cadets from a maritime vocational institution. Data were collected through a Contextual Mathematical Literacy Test (CMLT) developed using authentic maritime operational tasks. Descriptive and non-parametric analyses examined proficiency distributions and group differences. The findings indicate that most cadets perform at low to medium proficiency levels, highlighting the need for context-based and PISA-oriented mathematics instruction aligned with professional maritime demands.

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1. Introduction

The maritime industry operates in an environment characterized by high complexity, dynamic conditions, and strict safety requirements. Modern maritime operations depend heavily on accurate navigation, operational planning, and real-time decision-making, all of which require strong quantitative reasoning. Mathematical competence is therefore essential for maritime professionals to ensure operational efficiency and safety, particularly in situations involving uncertainty and risk (Akakpo, 2016; Veloo et al., 2015). Recent international

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evidence further indicates that many education systems continue to face challenges in developing learners' mathematical literacy, especially in tasks requiring contextual reasoning and higher-order thinking (OECD, 2023b).

Mathematical literacy extends beyond procedural knowledge of formulas and algorithms. It involves the ability to formulate, apply, and interpret mathematics in real-world contexts through reasoning, modeling, and critical reflection (De Lange, 2003; Niss & Jablonka, 2014; Stacey, 2011). In maritime practice, this competence is reflected in tasks such as route optimization, fuel consumption estimation, stability assessment, and interpretation of navigational data (Maternová et al., 2023; Stanivuk et al., 2017). Previous studies indicate that insufficient mathematical reasoning may contribute to operational inefficiencies and increase the likelihood of human error in safety-critical maritime environments (Veloo et al., 2015).

From an educational perspective, the Organisation for Economic Co-operation and Development (OECD) conceptualizes mathematical literacy through the Programme for International Student Assessment (PISA) framework, which defines mathematical literacy as an individual's capacity to formulate, employ, and interpret mathematics in a variety of real-life contexts (OECD, 2019). The framework classifies learners' competencies into progressive proficiency levels, ranging from basic procedural skills to advanced abstraction, modeling, and reflective reasoning (OECD, 2023a). While the PISA framework has been widely applied in general education research, its application in maritime vocational education particularly among nautical cadets whose professional responsibilities require precise navigation, operational planning, and safety critical decision making remains limited.

In this study, the term *contextual mathematical literacy* refers specifically to the ability of nautical cadets to formulate, apply, and interpret mathematical concepts within authentic maritime work-related situations. While the PISA framework conceptualizes mathematical literacy in broad real-world contexts (OECD, 2019, 2023a), this research narrows the scope to domain-specific vocational contexts by embedding mathematical tasks in realistic maritime operational scenarios, such as navigation planning, fuel estimation, stability calculations, and risk assessment. Therefore, contextual mathematical literacy in this study does not merely indicate general task-based problem solving, but reflects the application of mathematical reasoning within simulated professional maritime settings.

In maritime vocational education, mathematics instruction is often delivered in a predominantly theoretical manner, with limited integration of authentic operational contexts. As a result, cadets may demonstrate adequate performance in classroom-based assessments but encounter difficulties when applying mathematical knowledge to real maritime situations, such as navigation planning and operational decision-making (Karahalil et al., 2024; Sohaimi et al., 2022). This misalignment between instructional practices and professional demands reflects a broader challenge in vocational education, where mathematics learning is frequently disconnected from workplace contexts, limiting the development of transferable and context-based competencies (Grootenboer et al., 2022; Sappa & Aprea, 2014; Wheelahan, 2015).

Preliminary classroom observations and informal academic evaluations conducted at the participating institution indicated that many cadets experience difficulties when solving non-routine and context-based mathematical problems.

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Although cadets generally perform adequately on routine computational tasks, they often struggle to interpret contextual information, integrate multiple variables, and justify their reasoning in maritime-related problem scenarios. Similar challenges have been reported in vocational and applied education settings, where students demonstrate procedural fluency but limited conceptual understanding and problem-solving flexibility (She et al., 2018; Sohaimi et al., 2022). These observations suggest a potential gap between mathematical instruction and the cognitive demands of maritime professional practice.

Existing studies in maritime education have primarily focused on curriculum design, professional standards, and competency frameworks. However, empirical investigations that directly assess cadets' contextual mathematical literacy using internationally recognized frameworks are still scarce (Stanivuk et al., 2017). In particular, limited attention has been given to profiling cadets' mathematical literacy across PISA proficiency levels when confronted with authentic maritime problem scenarios (Novitasari et al., 2024). This lack of empirical evidence restricts educators' ability to design targeted instructional strategies to address specific weaknesses in cadets' mathematical reasoning.

To address this gap, the present study examines the contextual mathematical literacy of nautical cadets using the PISA proficiency framework with assessment tasks adapted to authentic maritime vocational contexts. This study contributes empirical evidence on cadets' mathematical literacy in authentic maritime contexts, extends the application of the PISA framework to vocational maritime education, and offers practical implications for the development of context-based and PISA-oriented mathematics instruction.

2. Method

This study used a quantitative descriptive design to describe the level of contextual mathematical literacy among nautical cadets based on the Programme for International Student Assessment (PISA) framework. The participants were 23 cadets enrolled in the Diploma III Nautical Studies Program at Politeknik Pelayaran Surabaya, Indonesia. Participants were selected using purposive sampling to ensure that all cadets had completed basic mathematics courses relevant to maritime applications. This design was chosen to obtain a clear and focused picture of cadets' mathematical literacy in a vocational maritime setting.

Data were collected using a Contextual Mathematical Literacy Test (CMLT) designed according to PISA mathematical literacy principles and adapted to real maritime situations. All test items were designed to simulate authentic maritime operational conditions to ensure alignment between PISA proficiency descriptors and professional maritime cognitive demands. The test consisted of five open-ended questions related to navigation calculations, route planning, and operational decision-making. These questions required cadets to apply mathematics in realistic maritime contexts rather than relying solely on routine calculations. Cadets' answers were scored using a rubric aligned with adapted PISA proficiency levels, ranging from Level 1c (lowest) to Level 6 (highest). The indicators used to classify each level of mathematical literacy are presented in Table 1.

Table 1. Adapted Mathematical Literacy Ability Level Indicators Based on the PISA Framework (OECD, 2023a)

E-ISSN: 2656-5846
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Level	Indicator
6	a. Cadets can solve abstract problems and demonstrate creativity and flexible thinking to develop solutions; b. Cadets can connect various sources of information and representations, including using simulations or spreadsheets effectively as part of a problem-solving solution; c. Cadets are able to think critically and have mastery of symbolic and formal mathematical operations, and are able to communicate clearly.
5	a. Cadets can develop and work with models for complex situations, identify or apply constraints, and determine assumptions; b. Cadets can apply systematic and planned problem-solving strategies to handle more challenging tasks; c. Cadets can reflect on and consider mathematical results by paying attention to real-world contexts.
4	a. Cadets can work effectively with explicit models for complex concrete situations, as well as demonstrate the ability to work with undefined models using more sophisticated computational thinking approaches; b. Cadets can select and integrate different information representations, and relate them directly to aspects of real-world situations; c. Cadets can construct and communicate explanations and arguments based on their interpretation, reasoning and methodology.
3	a. Cadets can design solution strategies or sequential decision making or flexibility in understanding familiar concepts; b. Cadets are able to complete tasks that require several different but routine calculations that are not all clearly defined in the problem statement; c. Cadets can use spatial visualization as part of a solution strategy or determine how to use simulation to collect appropriate data for the task.
2	a. Cadets can recognize situations where they need to design a simple strategy to solve a problem, including running a live simulation involving one variable as part of their solution strategy; b. Cadets can extract relevant information from one or more sources using slightly more complex modes of representation; c. Cadets are able to interpret the results literally.
1a	a. Cadets can answer questions involving simple contexts where all the required information is available, and the question is clearly defined; b. Cadets are able to carry out simple and routine procedures according to direct instructions in explicit situations to solve a problem; c. Cadets can apply basic algorithms, formulas, procedures, or conventions to solve problems.
1b	a. Cadets can respond to questions involving an easily understood context in which all necessary information is clearly provided and are able to recognize when some information is irrelevant; b. Cadets are able to perform simple calculations with whole numbers, following clear instructions.
1c	a. Cadets can respond to questions involving an easily understood context where all relevant information is clearly provided; b. Cadets are able to explain procedures or operational steps based on clear instructions.

In addition to proficiency levels, the CMLT was developed to reflect the three

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core mathematical processes defined in the PISA framework: formulating, employing, and interpreting (OECD, 2019, 2023a). Each item required cadets to translate maritime problem situations into mathematical representations (formulate), apply relevant procedures or models (employ), and interpret the results in relation to operational decision-making (interpret). The assigned proficiency levels (Levels 1c–6) represent increasing complexity and integration of these processes.

For analytical purposes, the six PISA proficiency levels were further grouped into three broader analytical categories to facilitate statistical comparison across performance levels. The categorization was based on the conceptual progression defined in the PISA framework rather than statistical quartile distribution, ensuring theoretical consistency with internationally recognized proficiency standards. The analytical grouping structure is presented in Table 2.

Table 2. Grouping of PISA Proficiency Levels into Analytical Categories

Analytical Category	PISA Proficiency Levels	Description of Cognitive Characteristics
Low Proficiency	Level 1c – Level 2	Ability limited to routine procedures, direct information use, and basic interpretation in clearly structured situations
Medium Proficiency	Level 3 – Level 4	Ability to apply mathematical reasoning in moderately complex contexts, integrate information, and construct explanations
High Proficiency	Level 5 – Level 6	Ability to model complex situations, think abstractly, evaluate solutions critically, and reflect on results

Data analysis was conducted in two stages: descriptive and inferential analysis. First, descriptive statistics were used to summarize cadets' contextual mathematical literacy levels, including the calculation of mean scores, frequency distributions, and percentage distributions across PISA proficiency levels. These analyses were used to provide an overall profile of cadets' literacy performance.

Prior to inferential testing, assumption tests were performed. The Shapiro–Wilk test was used to examine the normality of score distribution, while Levene's test was conducted to assess the homogeneity of variance across literacy groups. Because the data were not normally distributed, non-parametric statistical procedures were applied. Differences in mathematical literacy scores among the low, medium, and high proficiency categories were analyzed using the Kruskal-Wallis test. When significant differences were detected, pairwise comparisons were conducted using the Mann–Whitney U test to identify specific group differences.

3. Results and Discussion

3.1. Results

The analysis of contextual mathematical literacy revealed statistically significant variation across proficiency categories within the sample of nautical cadets, as indicated by the non-parametric tests based on the PISA framework. Overall, the results show that the distribution of performance was concentrated at lower

E-ISSN: 2656-5846
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proficiency levels when cadets solved problems situated in maritime contexts.

3.1.1. Distribution of Mathematical Literacy Levels

As shown in Table 3, more than half of the cadets (n = 13; 56.52%) were classified in the low literacy category, corresponding to PISA Levels 1 and 2.. Cadets in this category were generally able to perform simple and routine calculations when instructions were explicit. However, they experienced difficulties when problems required interpretation of contextual information, consideration of multiple variables, or application of mathematical reasoning to unfamiliar maritime situations.

Approximately 8 cadets (34.78%) reached the medium literacy category (PISA Levels 3 and 4). These cadets were able to apply appropriate mathematical procedures to contextualized problems, such as basic navigation planning and operational calculations. Nevertheless, their problem-solving approaches were predominantly procedural, with limited evidence of deeper reasoning or mathematical justification.

Only 2 cadets (8.70%) achieved high mathematical literacy levels (PISA Levels 5 and 6). Cadets in this category demonstrated the ability to analyze complex maritime scenarios, construct mathematical models, evaluate alternative solution strategies, and justify their reasoning. Their responses reflected greater flexibility and higher-order reasoning in addressing non-routine maritime problems.

Table 3. Distribution of Contextual Mathematical Literacy Levels Based on the PISA Framework

PISA Proficiency Level	Category	Frequency (n)	Percentage (%)
Level 6	High	1	4.35
Level 5	High	1	4.35
Level 4	Medium	3	13.04
Level 3	Medium	5	21.74
Level 2	Low	6	26.09
Level 1a	Low	4	17.39
Level 1b	Low	2	8.70
Level 1c	Low	1	4.35
Total		**23**	**100**

3.1.2. Assumption Testing and Inferential Analysis

Before conducting inferential analysis, assumption testing was performed. The Shapiro–Wilk normality test showed that the distribution of contextual mathematical literacy scores was not normal ($W = 0.891$, $p = 0.018$). In contrast, Levene's test indicated that the variance of scores across literacy groups was comparable ($F = 1.124$, $p = 0.342$).

Given the non-normal distribution of the data, non-parametric statistical procedures were applied. As shown in Table 4, the Kruskal–Wallis test revealed a statistically significant difference in contextual mathematical literacy scores among the low, medium, and high proficiency groups ($H = 15.276$, $df = 2$, $p = 0.001$), indicating meaningful differences in performance across literacy categories.

Subsequent pairwise comparisons using the Mann–Whitney U test confirmed statistically significant differences between all literacy groups. Significant differences were found between the low and medium proficiency groups ($U = 18.50$, $p = 0.004$), between the low and high proficiency groups ($U = 2.00$, $p = 0.001$),

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and between the medium and high proficiency groups (U = 5.00, p = 0.012). These findings demonstrate consistent performance gaps in contextual mathematical literacy across all proficiency levels.

Table 4. Summary of Statistical Test Results

Statistical Test	Statistic	Value	p-value
Shapiro–Wilk Normality Test	W	0.891	0.018
Levene's Test for Homogeneity	F	1.124	0.342
Kruskal–Wallis Test	H (df = 2)	15.276	0.001
Mann–Whitney U (Low–Medium)	U	18.50	0.004
Mann–Whitney U (Low–High)	U	2.00	0.001
Mann–Whitney U (Medium-High)	U	5.00	0.012

3.2. Discussion

The findings of this study indicate that the contextual mathematical literacy of nautical cadets is predominantly situated at low to medium proficiency levels. As shown in Table 3, most cadets were classified within PISA Levels 1 and 2, suggesting that their performance was largely limited to routine procedures and direct information use. This pattern aligns with previous research in vocational and applied education contexts, where learners often demonstrate procedural fluency but limited higher-order reasoning when mathematical instruction emphasizes computation over contextual understanding (She et al., 2018; Sohaimi et al., 2022).

The relatively small proportion of cadets achieving high proficiency levels (PISA Levels 5 and 6) indicates a substantial gap between current mathematical performance and the cognitive demands of maritime professional practice. Maritime operations require the ability to interpret contextual data, integrate multiple variables, and evaluate alternative solutions in dynamic environments. In this study, cadets classified at these levels demonstrated such competencies by systematically identifying relevant operational variables, constructing appropriate mathematical models, and providing justified interpretations of their results. For instance, in navigation-related tasks, high-proficiency cadets explicitly incorporated distance, fuel consumption rates, and time constraints into structured calculations and explained the operational implications of their chosen strategies. Prior studies have highlighted that insufficient mathematical literacy may constrain professionals' decision-making accuracy and increase the risk of operational errors in safety-critical settings (Akakpo, 2016; Maternová et al., 2023; Veloo et al., 2015).

The statistically significant differences identified through the Kruskal–Wallis and Mann–Whitney U tests further confirm that mathematical literacy develops progressively across proficiency levels rather than as a uniform skill. This finding is consistent with the PISA framework, which conceptualizes mathematical literacy as a continuum of increasingly complex competencies involving reasoning, modelling, and reflective thinking (OECD, 2019, 2023a).

From an educational perspective, the dominance of lower proficiency levels suggests a misalignment between instructional practices and professional maritime demands. When mathematics is taught primarily through routine procedural exercises without sufficient contextual integration, cadets may struggle to apply their knowledge in operational situations. Integrating maritime-relevant problem scenarios within instruction can support the development of flexible

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reasoning, modelling competence, and adaptive expertise required for real-world problem solving (Schukajlow et al., 2024). Such approaches also enhance learners' ability to connect mathematical concepts with authentic applications and engage more effectively with complex tasks (Boaler, 2016; Hong et al., 2020; Jablonka, 2003; Karahalil et al., 2024; Ojose, 2011).

Assessment practices likewise influence learning outcomes. Evaluations that focus exclusively on procedural correctness may not adequately capture reasoning quality or contextual interpretation. Incorporating open-ended and context-based assessment formats, aligned with PISA-oriented principles, can provide more meaningful insights into cadets' mathematical literacy and professional readiness (Brookhart, 2010).

Despite its contributions, this study has several limitations. The relatively small sample size and focus on a single maritime vocational institution may limit generalizability. In addition, the cross-sectional design does not capture developmental changes over time. Future research should involve larger and more diverse samples, apply longitudinal or experimental designs, and examine the effectiveness of context-based instructional interventions in strengthening mathematical literacy among maritime cadets. Comparative studies across institutions and countries would further enrich understanding of contextual mathematical literacy in maritime education (OECD, 2023a).

Overall, this study contributes by extending the application of the PISA mathematical literacy framework to a rarely researched maritime vocational context and by empirically profiling the contextual mathematical competencies of nautical cadets.

4. Conclusions

This study examined the contextual mathematical literacy of nautical cadets using an adapted PISA proficiency framework embedded in maritime-related scenarios. The findings reveal that most cadets are concentrated at the lower to medium levels of mathematical literacy, indicating that their competencies remain largely procedural and context-limited. Only a small proportion demonstrated advanced modelling and reflective reasoning skills aligned with higher PISA proficiency levels.

Rather than representing isolated categories, the low, medium, and high classifications reflect a progressive continuum of mathematical development. The dominance of lower proficiency levels suggests that many cadets have not yet transitioned toward the higher-order reasoning and modelling competencies required in complex maritime operations. These findings highlight the need for instructional approaches that support gradual progression from procedural fluency to contextualized, adaptive problem solving.

Overall, this study contributes empirical evidence on contextual mathematical literacy within maritime vocational education and underscores the importance of aligning mathematics instruction with authentic professional demands.

Author Contributions

The first author was responsible for the conceptualization of the study, research design, methodology development, data curation, formal data analysis, and preparation of the original manuscript draft. The second author, contributed through academic supervision, validation of the research process and findings,

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conducted the preliminary data analysis, and performed critical review and editing of the manuscript. The third author, assisted in data collection, supported data organization, and contributed to the manuscript writing under the supervision of the first and second authors.

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Declaration of Competing Interest
The authors declare no competing interests.

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Enhancing Students' Mathematical Problem-Solving Skills through Problem-Based Learning in Systems of Linear Equations in Two Variables at Junior High School Level

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ABSTRACT (11 pt)

This study investigated the effect of Problem-Based Learning (PBL) on students' mathematical problem-solving skills in Systems of Linear Equations in Two Variables (SPLDV), a contextual algebra topic that has received limited empirical attention in Indonesian junior high schools. Using a posttest-only control group design, the study involved 63 Grade VIII students of State Junior High School 2 Ngoro. The experimental group learned through PBL, while the control group received conventional instruction. The experimental group achieved a significantly higher posttest mean (78.12) than the control group ($p < 0.05$). The effect size (Hedges' $g = 1.14$) indicated a very high effect, suggesting that PBL is particularly effective for improving students' problem-solving skills in contextual SPLDV learning.

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1. Introduction

Education plays a strategic role in improving the quality of human resources, as it is regarded as a long-term investment that directly contributes to the development of individual competence, productivity, and competitiveness in responding to global dynamics (Becker, 1993). The quality of a nation's human resources is largely determined by the effectiveness of its educational processes, particularly in shaping students' thinking abilities, decision-making skills, and problem-solving competencies required in real-life situations (OECD, 2018). Therefore, improving educational quality has become a crucial strategy in addressing global challenges, which requires learning processes that focus not only on knowledge acquisition

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but also on the development of higher-order thinking skills.

Mathematics is a fundamental subject taught from elementary school to higher education and plays an essential role in the advancement of science and technology. The success of mathematics learning is not merely measured by students' skills to memorize formulas, but also by their skills to understand concepts and solve mathematical problems accurately (Sumiati & Agustini, 2020). Learning that emphasizes factual knowledge alone is considered insufficient to prepare students to face complex and contextual problems (Trilling & Fadel, 2009). Consequently, mathematics instruction should be directed toward the development of higher-order thinking skills, including the abilities to analyze, evaluate, and systematically solve problems (Zohar & Dori, 2003).

In the context of mathematics education, problem-solving skills is one of the core competencies that students must master. Mathematical problem solving not only reflects students' procedural skills but also indicates the depth of conceptual understanding and the skills to connect mathematics with real-life situations (Polya, 1973). Students with strong problem-solving skills are generally able to understand problems, design appropriate solution strategies, and evaluate the obtained solutions logically and reflectively.

One mathematical topic that requires strong problem-solving skills is the System of Linear Equations in Two Variables. This topic is a core subject at the junior high school level and is closely related to modeling real-life situations into mathematical representations. However, research by Cuong & Tien-Trung (2025) indicates that students often experience difficulties in connecting algebraic concepts with contextual problems, which negatively affects their mathematical problem-solving skills. Therefore, mastery of System of Linear Equations in Two Variables concepts requires well-developed problem-solving skills so that students are able to relate mathematical concepts to contextual situations effectively (Mardliyah et al., 2024).

Based on preliminary classroom observations, students' mathematical problem-solving skills in SPLDV was found to be relatively low. Students experienced difficulties in understanding problem statements, identifying variables, and designing appropriate solution strategies. Classroom practices were still dominated by conventional teacher-centered instruction, resulting in limited student engagement during learning activities (Zulkarnain, 2023). Notably, SPLDV learning tasks are predominantly presented as contextual word problems derived from everyday situations, which require students to translate real-world scenarios into algebraic models. This inherent contextual nature makes SPLDV particularly well-suited to Problem-Based Learning, which is grounded in learning through authentic, real-world problems.

Such learning practices limit students' involvement in constructing and applying problem-solving strategies. Learning that is predominantly procedural tends to restrict students' cognitive engagement, resulting in limited opportunities to develop critical and reflective thinking skills (Hattie, 2020). This condition highlights the need for learning approaches that actively engage students in higher-level thinking processes. Previous studies by Abdullah & Munawwaroh (2024) revealed that classroom learning is still largely dominated by teachers, which reduces students' participation in discussion, exploration, and problem-solving activities, thereby contributing to low mathematical problem-solving skills.

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Problem-Based Learning (PBL) is considered a relevant alternative learning model. PBL positions contextual problems as the starting point of learning, encouraging students to engage in exploration, discussion, and reflection in order to construct solutions (Hmelo-Silver, 2020). In PBL, learning problems are designed to involve students actively in questioning and problem-solving processes related to both content and context under investigation (Major & Mulvihill, 2018).

Although numerous studies have demonstrated the effectiveness of PBL, several recent findings suggest that its implementation does not always yield optimal results when students are not accustomed to active learning. This indicates that the effectiveness of PBL is highly dependent on the learning context, subject matter, and student characteristics (Hermawan & Siliwangi, 2024). Kappassova et al. (2025) further emphasized that PBL requires systematic instructional planning to help students adapt to active learning, particularly in abstract mathematical topics. Therefore, more specific investigations focusing on particular materials are necessary to obtain a more comprehensive understanding.

Based on the literature and classroom conditions, there remains a gap between the demand to develop students' mathematical problem-solving skills and classroom practices that continue to rely on conventional approaches. Moreover, studies that specifically examine the effect of PBL on students' mathematical problem-solving skills in SPLDV at the junior high school level remain limited. Accordingly, this study offers novelty by focusing on SPLDV and employing a quantitative experimental approach to examine the effect of PBL on students' mathematical problem-solving skills.

This study aims to: (1) describe the effect of the Problem-Based Learning model on Grade VIII students' mathematical problem-solving skills in SPLDV; and (2) determine the magnitude of the effect of the Problem-Based Learning model on students' mathematical problem-solving skills in SPLDV.

2. Method

This study employed a quantitative approach using a quasi-experimental posttest-only control design. The independent variable was the Problem-Based Learning (PBL) model, and the dependent variable was students' mathematical problem-solving skills. The population comprised all Grade VIII students of SMP Negeri 2 Ngoro in the 2025/2026 academic year (nine classes). The sample was selected using cluster random sampling, with Class VIII-H assigned as the experimental group (32 students) and Class VIII-I as the control group (31 students), resulting in a total sample of 63 students.

The treatment was conducted over two meetings in both groups on the topic of SPLDV. The experimental group was taught using PBL, while the control group received conventional instruction with the same learning objectives and content.

The research instrument was a mathematical problem-solving test consisting of three essay items (with the third item divided into two sub-items) developed based on Polya's indicators. Content validity was established through expert judgment provided by one mathematics education lecturer and one junior high school mathematics teacher. Data were analyzed using prerequisite tests (normality and homogeneity) followed by an independent samples t-test.

3. Results and Discussion

The descriptive statistics of posttest scores indicate differences in students'

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mathematical problem-solving skills between the experimental and control groups. Based on the descriptive statistical analysis presented in Table 1, the results indicated that the mean posttest score of the control class was 66.13, while the experimental class achieved a higher mean posttest score of 78.12. The descriptive analysis shows higher mean posttest scores in the experimental group compared to the control group. Subsequently, normality and homogeneity tests were conducted as prerequisite analyses.

Table 1. Descriptive Statistics of Posttest Scores

Group	N	Minimun	Maximum	Mean	Std. Deviation
Experimental Posttest	32	60	100	78.12	1.862
Control Posttest	31	40	85	66.13	1.865
Valid N (listwise)	63				

3.1. Normality Test

The normality test results of Table 2 indicate that the posttest data on students' mathematical problem-solving skills in both the experimental and control groups were normally distributed. In the experimental group, the Kolmogorov–Smirnov significance value was 0.072 and the Shapiro–Wilk significance value was 0.217. In the control group, the Kolmogorov–Smirnov significance value was 0.165 and the Shapiro–Wilk significance value was 0.061. All significance values exceeded 0.05, indicating that the data in both groups met the normality assumption and were appropriate for subsequent parametric analysis.

Table 2. Normality Test Result

Test	Group	Kolmogorof-Smirnov			Shapiro-wilk		
		Statistic	Df	Sig.	Statistic	df	Sig.
Mathematical Problem-Solving Skills	Experimental Posttest	.148	32	.072	.956	32	.217
	Control Posttest	.134	31	.165	.935	31	.061

3.2. Homogeneity Test

The homogeneity test results in Table 3 show that the variances of students' mathematical problem-solving scores in both the experimental and control groups were comparable. The Levene's test yielded a significance score of 0.814, indicating no substantial difference in score variskills between the two groups. This finding indicates that the score dispersion in both the experimental and control classes was generally analogous, signifying equivalent levels of score variation in both groups. Consequently, the condition of homogeneity of variances was fulfilled, allowing for the suitable comparison of the two groups utilizing parametric statistical analysis in the wnsuring hypothesis testing.

Tabel 3. Test of Homogeneity of Variances

Test	Levene Statistic	df1	df2	Sig.
Mathematical Problem-Solving Skills	.056	1	61	.814

3.3. Hypothesis Test

Hypothesis testing in this study was conducted using an independent samples t-

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test, which was applied to compare the mathematical problem-solving skills of students taught using the Problem-Based Learning model and those taught using conventional learning methods. As presented in Table 4, the independent samples t-test revealed a statistically significant difference in students' mathematical problem-solving scores between the experimental and control groups ($p < .001$). The mean difference between the two groups was 11.996 points, with the experimental group obtaining higher posttest scores than the control group. This finding indicates that students who learned through Problem-Based Learning achieved substantially higher mathematical problem-solving performance than those who learned through conventional instruction in the topic of Systems of Linear Equations in Two Variables.

Table 4. Independent Sampel Test for Mathematical Problem-Solving Skills

	Levene's Test for Equality of Variances		**t-test for Equality of Means**						
								95% Confidence Interval of the Difference	
	F	**Sig.**	**t**	**df**	**Sig. (2-tailed)**	**Mean Difference**	**Std.Error Difference**	**Lower**	**Upper**
Equal Variances Assumed	.056	.814	4.551	61	.000	11.996	2.636	6.725	17.267
Equal Variances not Assumed			4.552	60.979	.000	11.996	2.635	6.727	17.265

These results suggest that engaging students in learning activities centered on contextual problems and collaborative problem-solving, as implemented in the Problem-Based Learning model, is associated with better mathematical problem-solving outcomes. The present findings are consistent with previous research by Arfiani & Rismen (2025) which reported significant improvements in students' mathematical problem-solving skills following the implementation of Problem-Based Learning compared to conventional teaching approaches. Together, these results strengthen the empirical evidence that Problem-Based Learning is an effective instructional approach for enhancing students' mathematical problem-solving skills, particularly in the context of Systems of Linear Equations in Two Variables at the junior high school level.

3.4. Effect Size

Furthermore, to determine the effect size, effect size is a method used to measure the magnitude of the influence of one variable on another, which is not affected by sample size. It describes the strength of the relationship or the difference between independent variables. In this study, effect size calculation is employed to identify the extent of the influence of the Problem-Based Learning model on students' mathematical problem-solving skills on the topic of Systems of Linear Equations in Two Variables (SPLDV) in Grade VIII.

Effect size is used to determine how strong the impact of the Problem-Based Learning model is on students' mathematical problem-solving skills in the SPLDV material for Grade VIII by applying the Hedges' formula. Before obtaining the

E-ISSN: 2656-5846
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Hedges' value, the researcher must first calculate the pooled standard deviation of both groups, which consist of the experimental class and the control class, namely,

$$SD^*_{pooled} = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{(32 - 1)(10.530)^2 + (31 - 1)10.385^2}{32 + 31 - 2}}$$

$$= \sqrt{109.39}$$

$$= 10.45$$

Therefore, the Hedges' value is obtained as follows,

$$Hedges'g = \frac{M_1 - M_2}{SD^*_{pooled}}$$

$$= \frac{78.12 - 66.13}{10.45}$$

$$= \frac{11.99}{10.45}$$

$$= 1.14$$

Interpretation of Effect Size Using Classification (Tamur et al., 2020).

Table 5. Effect Size Category

Effect Size (ES)	Category
$-0{,}15 \leq ES \leq 0{,}15$	No level
$0{,}15 < ES \leq 0{,}40$	Low level
$0{,}40 < ES \leq 0{,}75$	Moderate level
$0{,}75 < ES \leq 1{,}10$	High level
$1{,}10 < ES \leq 1{,}45$	Very high level
$ES \geq 1{,}45$	Very good level

Based on the classification proposed by Tamur et al. (2020) the Hedges' g value of 1.14 falls into the very high category. This finding indicates that the impact of the Problem-Based Learning (PBL) model on students' mathematical problem-solving skills is not only statistically significant but also substantial in practical terms. In classroom practice, this means that the performance gap between students taught using PBL and those taught using conventional instruction is meaningful and observable.

The strong effect of PBL observed in this study can be attributed to the characteristics of PBL, which emphasize learning through contextual and real-world problems. Such learning activities encourage students to actively interpret problem situations, construct mathematical models, and develop systematic

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solution strategies, thereby supporting deeper conceptual understanding and more effective problem-solving processes (Ozer & Sezer, 2021). Learning through contextual problems also facilitates meaningful learning and conceptual transfer, enabling students to better apply mathematical concepts to real-life situations (Sungur & Tekkaya, 2021). Previous findings further indicate that PBL is associated with improvements in students' problem-solving performance and conceptual understanding (Tarhan & Ayyildiz, 2020).

Based on these findings, several practical implications can be drawn. Mathematics teachers are encouraged to integrate Problem-Based Learning, particularly when teaching topics that involve contextual word problems such as Systems of Linear Equations in Two Variables. Teachers may design learning activities that begin with real-life problem situations and guide students to collaboratively explore solution strategies. For future research, longer intervention periods and the inclusion of different mathematical topics are recommended to further examine the consistency of the PBL effect on students' mathematical problem-solving skills.

4. Conclusions

This study investigated the effect of the Problem-Based Learning (PBL) model on Grade VIII students' mathematical problem-solving skills in the topic of Systems of Linear Equations in Two Variables. The findings indicate that students who were taught using the PBL model achieved significantly higher posttest scores than those who received conventional instruction. This demonstrates that the application of PBL leads to better learning outcomes in terms of students' mathematical problem-solving skills.

Furthermore, the magnitude of the effect was classified as very high, indicating that the impact of the PBL model is substantial in practical classroom contexts. These results suggest that the implementation of Problem-Based Learning provides meaningful benefits for improving students' mathematical problem-solving skills. Therefore, PBL can be considered an effective instructional approach for mathematics learning at the junior high school level, particularly for topics that involve contextual problem situations such as Systems of Linear Equations in Two Variables.

Author Contributions

The first author was responsible for conceptualizing the study, developing the research methodology, conducting the investigation, collecting and analyzing the data, and writing the initial draft of the manuscript. The second author contributed by providing research supervision, validating the research findings, and reviewing and revising the manuscript.

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Declaration of Competing Interest

The authors declare that there is no conflict of interest related to this research.

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Estimation of Dengue Fever Transmission Model in West Java Using the Ensemble Kalman Filter Method

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ABSTRACT

Dengue Fever Transmission is a serious, potentially fatal infectious if unmanaged disease. It caused by the dengue virus, transmitted by Aedes aegypti and Aedes albopictus mosquitoes. In West Java, cases remain high and fluctuate significantly, requiring precise mathematical modeling to describe transmission dynamics. This is the first study applying Ensemble Kalman Filter (EnKF) in West Java to estimate the SEIR-SI model, compared to Unscented Kalman Filter (UKF), using dengue fever transmission case data from 2010–2023. Performance was assessed via Mean Absolute Percentage Error (MAPE) for the infected human population (I_h), showing EnKF's superior accuracy (2.4%) over UKF (7.8%). EnKF effectively estimates hard-to-measure epidemiological variables and this study can support government prediction-based dengue fever transmission control policies.

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1. Introduction

Dengue fever transmission is an infectious disease caused by the dengue virus and transmitted through the bites of *Aedes mosquitoes*, primarily *Aedes aegypti* and *Aedes albopictus* (Messina et al., 2020) . The dengue virus belongs to the genus *Flavivirus* of the *Flaviviridae* family and consists of four distinct serotypes, namely DEN-1, DEN-2, DEN-3, and DEN-4 (World Health Organization, 2024)(Papa et al., 2024). Dengue infection is characterized by an incubation period of approximately 4–10 days, followed by clinical manifestations such as persistent high fever, skin rash,

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gastrointestinal symptoms, and a significant decrease in platelet count (Narinder, Ajeet, and Amar, 2022)(Namirimu and Kim, 2024). The critical phase typically occurs between the third and seventh day of illness and may lead to serious complications, including plasma leakage, severe bleeding, and dengue shock syndrome, which can be fatal if not properly managed (Kularatne et al., 2025).

Globally, dengue fever transmission remains a major public health concern, particularly in tropical and subtropical regions where more than half of the world's population is at risk of dengue fever transmission (World Health Organization, 2024)(Zhang et al., 2025). The life cycle and population dynamics of *Aedes aegypti* as the primary vector are strongly influenced by climatic factors such as temperature, humidity, and rainfall patterns (Liu et al., 2023). In the context of global climate change, extreme climate phenomena such as El Niño further exacerbate environmental conditions by increasing average temperatures and altering rainfall variability (Bolan et al., 2024). These conditions accelerate mosquito reproduction cycles and increase biting frequency, thereby elevating the risk of dengue fever transmission (Nasirian, 2024)(Awaludin, Prayitno, and Haq, 2023).

In Indonesia, dengue fever transmission has become an endemic disease with a noticeable increase in reported cases over recent years. Official records indicate that dengue cases rose from 73,518 cases with 705 deaths in 2021 to 143,000 cases with 1,236 deaths in 2022. By the 43rd week of 2024, the number of reported cases surged to 210,644, accompanied by 1,239 fatalities. The high incidence of dengue is closely associated with population density, particularly on Java Island. Provinces such as West Java, East Java, and Central Java consistently report the highest number of cases, with West Java recording the largest burden due to rapid urbanization, dense settlements, and limitations in vector control infrastructure (Kementerian Kesehatan Republik Indonesia, 2024).

The persistent increase in dengue cases highlights the need for a systematic understanding of the disease transmission dynamics (Zeng et al., 2023). Such understanding requires accurate estimation of state variables within a dynamic disease transmission model. Mathematical modeling has been widely employed to represent the spread of infectious diseases, especially for systems in which several components cannot be directly observed. Estimation techniques play a crucial role in inferring unobservable variables based on available observational data, making them particularly important for complex nonlinear epidemiological systems. Several studies have explored estimation approaches in infectious disease modeling. (Safitria, Arif, and Fatmawati, 2023) applied the Ensemble Kalman Filter (EnKF) to a nonlinear continuous malaria transmission model in Indonesia using cumulative case data from 2010–2022, incorporating forward finite difference discretization and linearization prior to assimilation; the results yielded a low RMSE of 0.005, indicating strong alignment with observational data, though the linearization step may constrain accuracy under extreme nonlinearity.

Other works have extended EnKF to dengue. For instance, (Cohnstaedt and Scoglio, 2021) proposed an SEIR-SEI-EnKF framework for dengue outbreak estimation and forecasting, showing improved performance over simpler models like SIR-EnKF or SEIR-EnKF in both simulations and real-world data. (Papageorgiou and Tsaklidis, 2022) developed a hybrid SEIHCRDV-UKF model for COVID-19 prediction, effectively estimating time-dependent parameters and unobserved states in complex epidemiological systems.

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Although these approaches have proven effective for outbreak prediction in endemic regions of China, similar applications to the dengue SEIR–SI model remain limited, particularly when using regional data from Indonesia such as West Java Province. Prior studies have largely focused on short- to medium-term forecasting rather than precise state variable estimation to directly inform control strategies (e.g., estimating unobserved latent infections, infected vectors, or recovered individuals). Moreover, direct comparisons between EnKF and the Unscented Kalman Filter (UKF) on local dengue datasets have received little attention, despite the methods' differing trade-offs in addressing nonlinearity, data noise, and computational demands across varying state dimensions.

This study bridges these gaps by applying EnKF to a tailored SEIR–SI dengue model in West Java, capitalizing on ensemble flexibility for more accurate state estimation in complex nonlinear systems with local observational data. A comparative analysis with UKF serves as validation to assess the relative effectiveness and precision of both methods in this context. The results of this study are expected to provide a strong quantitative basis for health authorities in establishing more consistent and data-based dengue control and prevention strategies.

2. Method

The methods employed in this study include SEIR–SI modeling for dengue transmission, discretization of the differential equations, and the application of EnKF and UKF for state variable estimation and model performance evaluation.

2.1. SEIRSI Model for Dengue Fever Transmission

Dengue fever transmission can be represented using various host–vector frameworks that describe how infections move between human and mosquito populations. Such formulations typically extend compartment-based models to incorporate biological processes, incubation periods, and vector dynamics, ensuring that the resulting system better reflects real epidemiological behavior. In this study, the dengue fever transmission model employed is an extension of the nonlinear host–vector model proposed by (Wijayanti and Fakhruddin, 2023).

A time-dependent host–vector model for dengue fever transmission was developed by incorporating an SEIR framework for the human population and an SI framework for the mosquito population. In this model, the human population is categorized into susceptible (S_h), exposed (E_h), infected (I_h), and recovered (R_h) groups, while the mosquito population is divided into susceptible (S_v) and infected (I_v) classes.

Time-varying transmission rates for both humans and vectors are applied to account for unobservable factors that influence the transmission dynamics. The transition processes between compartments are illustrated in Figure 1. Based on this formulation, a compartmental diagram was constructed to represent the spread of dengue fever transmission as follows.

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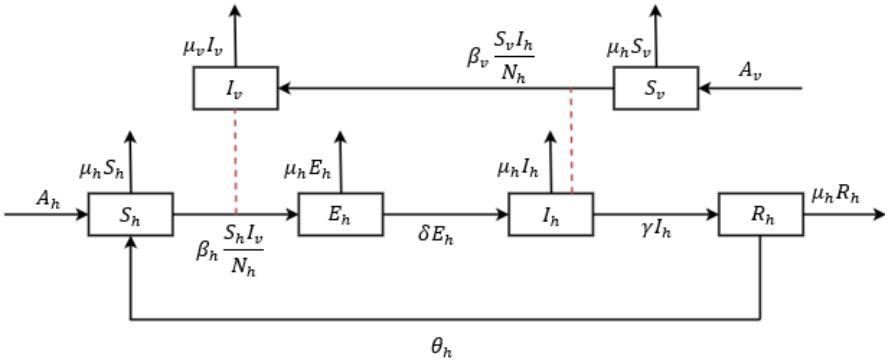


Figure 1. Compartment Diagram of the Dengue Fever Transmission Model

$$\frac{dS_h}{dt} = \Lambda_h - \beta_h \frac{S_h I_v}{N_h} - \mu_h S_h + \theta R_h \quad (1)$$

$$\frac{dE_h}{dt} = \beta_h \frac{S_h I_v}{N_h} - \delta E_h - \mu_h E_h \quad (2)$$

$$\frac{dI_h}{dt} = \delta E_h - \gamma I_h - \mu_h I_h \quad (3)$$

$$\frac{dR_h}{dt} = \gamma I_h - \mu_h R_h - \theta R_h \quad (4)$$

$$\frac{dS_v}{dt} = \Lambda_v - \beta_v \frac{S_v I_h}{N_h} - \mu_v S_v \quad (5)$$

$$\frac{dI_v}{dt} = \beta_v \frac{S_v I_h}{N_h} - \mu_v I_v \quad (6)$$

The parameters used in the system are defined as follows.

S_h : Human population susceptible to dengue transmissio
E_h : Human population exposed to the dengue virus but not yet infectious
I_h : Human population infected and capable of transmitting the virus
R_h : Recovered human population
S_v : *Aedes* mosquito population not yet infected and potentially contract the virus after contact with an infected human
I_v : Mosquito population infected and capable of transmitting the dengue virus throughout its life
N_h : Total human population
N_v : Total mosquito population
Λ_h : Human birth/growth rate
Λ_v : Mosquito birth/growth rate
μ_h : Natural human mortality (death) rate
μ_v : Natural mosquito mortality (death) rate
β_h : Transmission rate of the virus from infected mosquitoes to susceptible Humans
β_v : Transmission rate of the virus from infected humans to susceptible Mosquitoes
δ : Transition rate from exposed humans (E_h) to infected (I_h)
γ : Transition rate from infected humans (I_h) to recovered (R_h)
θ : Rate of waning immunity in humans

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2.2. Model Discretization

The dynamic model equations describing the spread of Dengue Fever are formulated as continuous differential equations. In order to perform variabel estimation, the differential equation model must be expressed in a discrete form; therefore, a discretization process is required (Villela, 2020). Model discretization can be carried out using several numerical methods, one of which is the forward finite difference method.

If $S = S(x)$ is expanded using a Taylor series, the following expressions are obtained,

$$S(x+h) = S(x) + h\frac{d}{dx}S(x) + \frac{h^2}{2!}\frac{d^2}{dx^2}S(x) + \cdots$$
$$S(x-h) = S(x) - h\frac{d}{dx}S(x) + \frac{h^2}{2!}\frac{d^2}{dx^2}S(x) + \cdots$$

In this study, the forward finite difference method is employed. The numerical scheme of the forward finite difference method is given by,

$$S(x+h) - S(x) = h\frac{d}{dx}S(x) + O(h^2)$$

Thus, the first-order derivative can be approximated as,

$$\frac{dS}{dx} \approx \frac{S(x+h) - S(x)}{h} \tag{7}$$

Equation (7) represents the forward finite difference approximation. Using finite difference notation with $u(x = kh)$, where $h = \Delta x$ and $k = 1,2,3, \ldots, n$, Equation (7) can be rewritten as (Listosari, 2018),

$$\frac{dS}{dx} = \frac{S^{k+1} - Sk}{\Delta x} \tag{8}$$

2.3. Ensemble Kalman Filter (EnKF)

The Ensemble Kalman Filter (EnKF) is a modified estimation method derived from the Kalman Filter algorithm that can be applied to both linear and nonlinear system models (Carrassi and Bocquet, 2018). The EnKF method was introduced by Evensen in 1994 (Tong, Gu, and Cheng 2024). The estimation process in the EnKF begins by generating a set of $N_e$ensembles with zero mean and constant covariance. These ensembles are generated randomly and are assumed to follow a normal distribution. Based on empirical studies, an adequate number of ensemble members typically ranges from 100 to 500. In the EnKF, error propagation does not require linearization of the model as in the Extended Kalman Filter (EKF). Instead, it is performed directly by independently integrating each ensemble member forward using the nonlinear model operator.

This approach makes the EnKF particularly well-suited for strongly nonlinear systems, as the error statistics are evolved using a Monte Carlo method through the ensemble members (Brajard et al., 2021). In general, the EnKF algorithm consists of two main stages, namely the prediction stage and the correction

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(update) stage (Roger, 2020). In the EnKF framework, the ensemble mean is computed prior to entering the prediction step. The general form of a nonlinear dynamic system in the EnKF can be expressed as follows,

$$x_{k+1} = f(x_k, u_k) + w_k$$

with a linear measurement $z_k \in R^p$, namely,

$$z_k = H_k x_k + v_k$$
$$x_0 \sim N(\bar{x}_0, P_{x0}); \; w_k \sim N(0, Q_k); \; v_k \sim N(0, R_k)$$

where H_k is a measurement matrix that shows which variables can be measured. The EnKF algorithm is presented in Table 1.

Table 1. The EnKF algorithm

System Model and Measurement Model	
	$x_{k+1} = f(x_k, u_k) + w_k, ; w_k \sim (0, Q_k)$ $z_k = H x_k + v_k, \quad ; v_k \sim (0, R_k)$
Initialization	Generate N ensembles based on the initial estimate $\hat{x}_0$, $x_{0,i} = [x_{0,1}, x_{0,2}, x_{0,3}, \dots, x_{0,N}]$ Determine the initial ensemble mean, $\hat{x}_0 = \frac{1}{N}\sum_{i=1}^{N} x_{0,i}$
Prediction Step	$\hat{x}_{\bar{k},i} = f(\hat{x}_{k-1}, u_{k-1}) + w_{k,i}, w_{k,i} \sim (0, Q_k)$ $\hat{x}_{\bar{k}} = \frac{1}{N}\sum_{i=1}^{N} (\hat{x}_{\bar{k},i})$ Error covariance $P_{\bar{k}} = \frac{1}{N-1}\sum_{i=1}^{N} (\hat{x}_{\bar{k},i} - \hat{x}_{\bar{k}})(\hat{x}_{\bar{k},i} - \hat{x}_{\bar{k}})^T$
Correction (Update) Step	$z_{k,i} = z_k + v_{k,i}, v_{k,i} \sim (0, R_k)$ Kalman gain, $K_k = P_k^- H^T (H P_k^- H^T + R_k)^{-1}$ State update, $\hat{x}_{k,i} = \hat{x}_{k,i}^- + K_k (z_{k,i} - H \hat{x}_{k,i}^-)$ $\hat{x}_k = \frac{1}{N}\sum_{i=1}^{N} \hat{x}_{k,i}$ Error covariance, $P_k = [I - K_k H] P_k^-$

(Roger, 2020)

2.4. Unscented Kalman Filter (UKF)

The Unscented Kalman Filter (UKF) is an extension of the Kalman Filter and is generally used to address nonlinear and more complex system problems. The concept involves generating several sampling points (Sigma Points) around the estimated condition based on their covariance (Singh, Mishra, and Chattopadhyay, 2024). These points are then propagated through a nonlinear mapping to obtain more accurate estimates of the mean and covariance of the

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resulting mapping (Munch, Rogers, and Sugihara, 2022).

Based on its application, the UKF method can be applied to variable estimation, parameter estimation, and dual estimation. The UKF was first introduced by Simon J. Julier and Jeffery K. Uhlmann in their paper entitled "A New Extension of the Kalman Filter to Nonlinear Systems" in 1997 (Jiang, Shi, and Moura, 2025). The UKF consists of two stages, similar to the KF: a prediction stage (time update) and a correction stage (measurement update). The UKF's function is to simplify the probability distribution approach for approximating nonlinear functions. The error estimates and covariances of the UKF are obtained using the Unscented Transformation. Unscented transformation is a method used to calculate the statistical value of a random variable that has undergone a nonlinear transformation (Li et al., 2025). The UKF algorithm is presented in Table 2

Table 2. The UKF algorithm

System Model and Measurement Model	
Initialization	$x_{k+1} = f(x_k, u_k) + w_k$ $z_k = H\,x_k + v_k$ $x_0 \sim N(\bar{x}_{0,} P_{x_0}), w_k \sim N(\bar{x}_{0,} Q_k), v_k \sim N(0, R_k)$ $\chi_{i\|k} = [\hat{\chi}_{i\|k} \quad \hat{\chi}_{i\|k} + \sqrt{(L+\lambda)P_{i\|k}} \quad \hat{\chi}_{i\|k} - \sqrt{(L+\lambda)P_{i\|k}}$
	Sigma point, $\chi_{i\|k} = [\hat{\chi}_{i\|k} \quad \hat{\chi}_{i\|k} + \sqrt{(L+\lambda)P_{i\|k}} \quad \hat{\chi}_{i\|k} - \sqrt{(L+\lambda)P_{i\|k}}$ $i = 1,2,\dots,2L$ and $k = 1,2,\dots,N$ where, $\gamma = \sqrt{L+\lambda}$ and $\lambda = \alpha^2(L+K) - L$ So that it is obtained, $\gamma = \sqrt{\alpha^2(L+K)} = \alpha\sqrt{L+K}$
Prediction Step	Estimate $\chi^f_{i\|k} = f(\chi_{i\|k})$ $\hat{x}^-_{k+1} = \sum_{i=0}^{2L} W_i^m \chi^f_{i\|k}$ Error covariance $P^-_{k+1} = \sum_{i=0}^{2L} W_i^c (\chi^f_{i\|k} - \hat{x}^-_{k+1})(\chi^f_{i\|k} - \hat{x}^-_{k+1})^T + Q$
Correction (Update) Step	Measurement model, $Y^f_{i\|k} = H(\chi^f_{i\|k})$ $\hat{y}^-_{k+1} = \sum_{i=0}^{2L} W_i^m Y^f_{i\|k}$ $P_{\hat{y}_{k+1}\hat{y}_{k+1}} = \sum_{i=0}^{2L} W_i^c (Y^f_{i\|k} - \hat{y}^-_{k+1})(Y^f_{i\|k} - \hat{y}^-_{k+1})^T + R$ Measurement and state covariance, $P_{\hat{x}_{k+1}\hat{y}_{k+1}} = \sum_{i=0}^{2L} W_i^c (\chi^f_{i\|k} - \hat{x}^-_{k+1})(Y^f_{i\|k} - \hat{y}^-_{k+1})^T$
	Kalman gain,

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System Model and Measurement Model
$K_{k+1} = P_{\hat{x}_{k+1}\hat{y}_{k+1}}\left(P_{\hat{y}_{k+1}\hat{y}_{k+1}}\right)^{-1}$
Estimate,
$\hat{x}_{k+1} = \hat{x}_{k+1}^{-} + K_{k+1}(y_k - H\hat{y}_{k+1}^{-})$
Error covariance,
$P_{k+1} = P_{k+1}^{-} - K_{k+1}P_{\hat{y}_{k+1}}\hat{y}_{k+1}K_{k+1}^{T}$

(Roger, 2020)

2.5. Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is a standard metric used to evaluate the accuracy of forecasting and estimation methods, expressed as the average absolute percentage error. MAPE is employed to assess the accuracy of predicted results by comparing them with observed values. Each observation period is indexed by t, where $t = 1,2,\ldots,n$, and $n$denotes the total number of observations. The actual value at period $t$is represented by X_t, while the forecasted value for the same period is denoted as $\hat{X}_t$. A smaller MAPE value indicates a higher level of prediction accuracy. The formula used to compute the MAPE value is given as follows (Hyndman and Athanasopoulos, 2021),

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - \hat{X}_t}{X_t}\right| \times 100\%$$

Table 3 presents the interpretation of estimation results using the MAPE to determine the level of accuracy of the obtained results.

Table 3. MAPE Accuracy Level

MAPE	Estimation Accuracy Level
$< 10\%$	Very Accurate
$10\% \leq MAPE < 20\%$	Good
$20\% \leq MAPE < 50\%$	Fair
$\geq 50\%$	Not Accurate

3. Results and Discussion

In this study, the SEIR–SI dengue fever transmission model is estimated using the EnkF method. The discretized SEIR–SI model employed in this research is formulated as a nonlinear system, as described below,

$$\begin{bmatrix} S_{h_{k+1}} \\ E_{h_{k+1}} \\ I_{h_{k+1}} \\ R_{h_{k+1}} \\ S_{v_{k+1}} \\ I_{v_{k+1}} \end{bmatrix} = \begin{bmatrix} \Delta t\left(\lambda_h - \frac{\beta_h}{N_h}S_{h_k}I_{v_k} - \mu_h S_{h_k} + \theta R_{h_k}\right) + S_{h_k} \\ \Delta t\left(\frac{\beta_h}{N_h}S_{h_k}I_{v_k} - \delta E_{h_k} - \mu_h E_{h_k}\right) + E_{h_k} \\ \Delta t\left(\delta E_{h_k} - \gamma I_{h_k} - \mu_h I_{h_k}\right) + I_{h_k} \\ \Delta t\left(\gamma I_{h_k} - \mu_h R_{h_k} - \theta R_{h_k}\right) + R_{h_k} \\ \Delta t\left(\lambda_v - \frac{\beta_v}{N_h}S_{v_k}I_{h_k} - \mu_v S_{v_k}\right) + S_{v_k} \\ \Delta t\left(\frac{\beta_v}{N_h}S_{v_k}I_{h_k} - \mu_v I_{v_k}\right) + I_{v_k} \end{bmatrix} + \begin{bmatrix} w_{1_k} \\ w_{2_k} \\ w_{3_k} \\ w_{4_k} \\ w_{5_k} \\ w_{6_k} \end{bmatrix} \quad (9)$$

In equation (9), w_k denotes the process noise vector, which captures model uncertainties and stochastic variations in dengue fever transmission dynamics. It

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is assumed to follow a zero-mean Gaussian distribution, $w_k \sim \mathcal{N}(0, Q)$, where Q is the process noise covariance matrix. The inclusion of process noise converts the deterministic SEIR–SI model into a stochastic state-space system, enabling EnKF and UKF to update state estimates by accounting for uncertainty in model evolution.

Tabel 4. Initial Values of Each Variable and Parameter

Variable	Initial Value	Parameter	Initial Value
S_h	43017127	λ_h	0.023757
E_h	6366	δ	0.125
I_h	3183	μ_h	0.000046
R_h	3151	θ_h	0.575
S_v	600000000	γ_h	0.328833
I_v	10000000	μ_v	0.0323
		β_h	0.75
		λ_v	0.5
		β_v	0.375

The data used in this study are secondary data consisting of the cumulative number of human populations infected with dengue fever transmission in West Java Province during the period 2010–2023, obtained from the Open Dengue website. The dataset comprises monthly data over 168 months, covering the period from January 2010 to December 2023. The simulation is conducted by assigning initial values to the system noise covariance matrix Q_k, namely $10^6, 10^4, 10^4, 10^3, 10^3$and 10^5, and the measurement noise covariance $R = 10^{-3}$. The initial values of the state variables are provided in Table 3. The measurement model used in this study is given as follows,

$$z_k = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_h \\ E_h \\ I_h \\ R_h \\ S_v \\ I_v \end{bmatrix} \tag{10}$$

In the observation model $z_k = Hx_k + v_k$, the term $v_k \sim \mathcal{N}(0, R)$represents measurement noise. In the implementation, this noise is not added as random values to the measurement data z_k at each step, instead, it is incorporated through the covariance matrix R during the Kalman update stage. The simulation results are as follows.

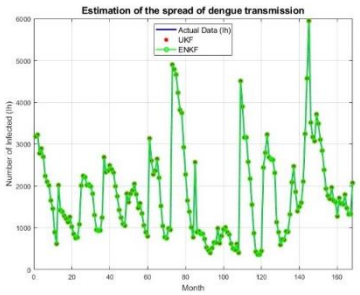


Figure 1. The Estimation Results for the Infected Human Population Capable of Transmitting the Virus

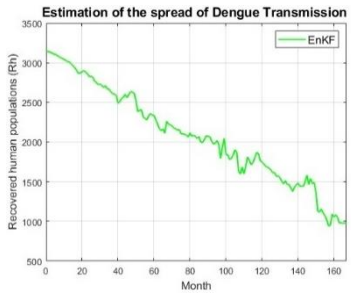


Figure 4. The Estimation Results for the Recovered Human Population

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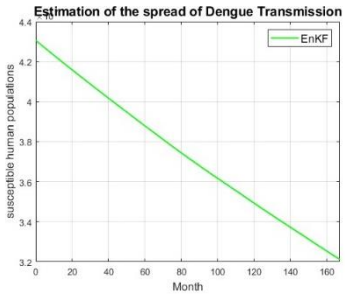


Figure 2. The Estimation Results for the Human Population Susceptible to Dengue Infection

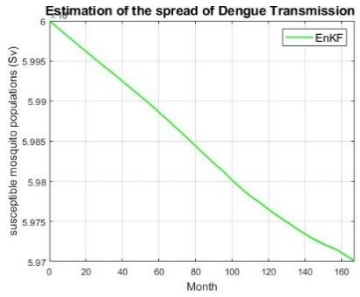


Figure 5. The Estimation Results for the *Aedes* Mosquito Population not yet Infected with the Dengue Virus

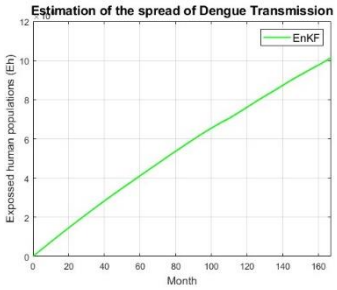


Figure 3. The Estimation Results for the Human Population Exposed to the Dengue Virus (Infected but not yet Infectious)

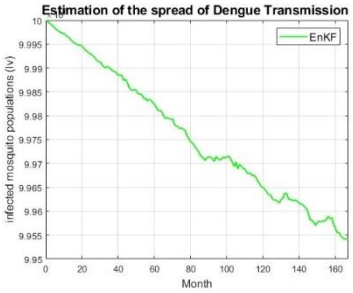


Figure 6. The Estimation Results for the Mosquito Population that has been Infected and Remains Capable of Transmitting the Dengue Virus throughout Its Lifetime

3.1. The Estimation Results for the Infected Human Population Capable of Transmitting the Virus

Figure 2 illustrates the estimation of the infected human population (I_h) in dengue fever transmission. The blue line represents the actual infected human population, obtained from the cumulative monthly dengue case data in West Java Province for the period 2010–2023. The green curve shows the estimated values generated using the EnKF which achieved a MAPE of 2.4%. Meanwhile, the red circles indicate the estimated values obtained using the Unscented Kalman Filter (UKF), with a MAPE of 7.8%.

These simulation results demonstrate that the EnKF provides a more suitable estimation method for the nonlinear dengue fever transmission model, as shown by its lower and more accurate MAPE value compared to the UKF. Based on the comparison presented in Figure 2, further estimation of other state variables using the EnKF can be performed. The estimated values for S_h, E_h, I_h, R_h, S_v, and I_v obtained using the EnKF are presented as follows.

3.2. The Estimation Results for the Human Population Susceptible to Dengue Infection

The simulation graph in Figure 3 presents the estimation of the susceptible human population (S_h) in the transmission of dengue fever transmission. The green curve represents the estimated values obtained using the EnKF method. The figure shows that the susceptible population exhibits a gradual decreasing trend on a monthly basis from early 2010 to the end of 2023. This decline is attributed to the

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transition of individuals into the infected class through the infection rate, as well as natural mortality.

3.3. The Estimation Results for the Human Population Exposed to the Dengue Virus (Infected but not yet Infectious)

The simulation graph in Figure 4 presents the estimation of the exposed human population (E_h), representing individuals who have been infected with the dengue virus but are not yet capable of transmitting it, in the transmission of dengue fever transmission. The green curve represents the estimated values obtained using the EnKF method. The figure shows that the exposed population exhibits a gradual increasing trend on a monthly basis from early 2010 to the end of 2023. This increase occurs because the number of exposed individuals continues to rise over the study period, primarily due to the transition of individuals from the susceptible class to the exposed class.

3.4. The Estimation Results for the Recovered Human Population

The simulation graph in Figure 5 presents the estimation of the recovered human population (R_h). The green curve represents the estimated values obtained using the EnKF method. The figure shows that the recovered population exhibits fluctuations over certain periods; however, overall, it demonstrates a decreasing monthly trend from early 2010 to the end of 2023. This decline is attributed to a significant increase in new infections, which leads to a reduction in the number of recovered individuals over time.

3.5. The Estimation Results for the Aedes Mosquito Population not yet Infected with the Dengue Virus

The simulation graph in Figure 6 presents the estimation of the susceptible *Aedes* mosquito population (S_v) in the transmission of dengue fever transmission. The green curve represents the estimated values obtained using the EnKF method. The figure shows that the susceptible mosquito population exhibits a decreasing trend from 2010 to 2023. This decline is attributed to the transition of mosquitoes from the susceptible class to the infected class as a result of dengue virus transmission, leading to a reduction in the number of susceptible mosquitoes over time.

3.6. The Estimation Results for the Mosquito Population that has been Infected and Remains Capable of Transmitting the Dengue Virus throughout Its Lifetime

The simulation graph in Figure 7 presents the estimation of the dengue virus–infected mosquito population (I_v) in the transmission of dengue fever transmission. The green curve represents the estimated values obtained using the EnKF method. The figure shows that the infected mosquito population exhibits a decreasing trend from 2010 to 2023. This decline is primarily attributed to natural mosquito mortality, which reduces the number of infected mosquitoes over time.

4. Conclusions

This study implemented the Ensemble Kalman Filter (EnKF) and the UKF within a nonlinear SEIR–SI model to estimate the state variables of dengue fever transmission using cumulative monthly case data from West Java Province during 2010–2023. The integration of the dynamic model with a linear measurement model enabled the estimation of both the infected human population (Ih) and other

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unobserved compartments. Simulation results show that EnKF achieved a MAPE of 2.4%, while UKF yielded 7.8%. Both methods fall within the high-accuracy category (MAPE < 10%); however, EnKF produced lower estimation error and demonstrated superior performance in reconstructing cumulative infection dynamics. This study is limited by the exclusion of environmental factors, population mobility, and spatial heterogeneity from the model structure. Nevertheless, the findings may support prediction-based dengue control policies by providing more measurable information on case dynamics. Future research may extend the model by incorporating external variables, such as climatological factors and population mobility, to represent dengue fever transmission dynamics more comprehensively.

Author Contributions

The first author was responsible for collecting, processing, and analyzing the data, as well as writing the scientific article. The second author contributed by offering feedback and suggestions to improve the study, its analysis, and the overall manuscript.

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Declaration of Competing Interest

There are no indications of potential conflicts of interest disclosed by the authors.

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The Effect of GeoGebra-Based Instruction on Students' Conceptual Understanding of Exponential Functions

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ABSTRACT

Exponential functions pose conceptual challenges for secondary students due to their abstract nature. This quasi-experimental study investigated the effect of GeoGebra-based instruction on Grade 10 students' conceptual understanding in two Indonesian senior high schools (N = 132). A validated four-item essay test (r > 0.219; Cronbach's Alpha = 0.71) was administered before and after instruction. Mann–Whitney U analysis showed a significant difference in posttest conceptual understanding between the GeoGebra and conventional groups ($p < .001$), with a moderate-to-large effect size (r = 0.44). These results provide classroom-based empirical evidence that dynamic visualization through GeoGebra strengthens students' conceptual understanding and supports its structured integration in conceptually demanding mathematics topics.

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1. Introduction

Exponential functions are a core component of the senior high school mathematics curriculum due to their wide-ranging applications in fields such as biology, economics, and computer science. Despite their relevance, students often struggle to understand exponential functions, particularly in interpreting rapid exponential growth and the shapes of graphs that represent such phenomena(Cardozo & Possamai, 2019)(Nunes et al., 2017). These difficulties are frequently attributed to students' tendency to focus on procedural mastery rather than conceptual understanding, thereby hindering their ability to relate exponential models to real-

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world contexts(Dorner & Ableitinger, 2022)(Hidayat et al., 2023)(Liburd & Jen, 2021).

Compounding these challenges is students' limited ability to visualize and interpret exponential graphs. Transitions from linear to exponential thinking require more sophisticated conceptual frameworks, which traditional instructional approaches often fail to adequately support(Iparraguirre-Villanueva et al., 2024) (Nunes et al., 2017). In Palu City, similar issues were observed at State Senior High School 2 Palu and State Senior High School 4 Palu. Based on preliminary classroom observations and informal discussions with mathematics teachers, the integration of digital technology—particularly dynamic mathematics software—into the teaching of exponential functions remained limited. GeoGebra, a dynamic mathematics software, had not yet been employed to facilitate student visualization or understanding of exponential growth patterns(Putra et al., 2021)(Weinhandl et al., 2020).

GeoGebra provides an avenue to enhance students' understanding of abstract mathematical concepts through the value of interactive, student-centered, and multi-representational learning environments. This approach is rooted in constructivist and mathematical representation theories. These theoretical underpinnings align well with GeoGebra's capabilities and its pedagogical application.

A growing body of research underscores GeoGebra's potential to support students' conceptual understanding through dynamic and interactive visual representations. By enabling real-time manipulation of mathematical objects, the software facilitates the coordination of graphical and algebraic representations, thereby strengthening representational fluency and deep structural comprehension(Aliyu et al., 2021)(Daulay et al., 2021)(Solvang & Haglund, 2021)(Wassie & Zergaw, 2019)(Zagoto et al., 2025). Within a constructivist framework, such interactive environments position learners as active meaning-makers, allowing conceptual understanding to emerge through exploration rather than procedural imitation. Increased motivation and participation reported in prior studies(Wahyuni et al., 2025) can thus be interpreted as consequential to, rather than independent from, deeper conceptual engagement.

Despite these documented affordances, the effectiveness of GeoGebra remains context-sensitive. Variations in instructional design, teacher expertise, and technological infrastructure contribute to inconsistent empirical outcomes(Albaladejo et al., 2015)(Dildabayeva & Zhaydakbayeva, 2024). More importantly, although GeoGebra has been extensively examined in geometry and general algebra contexts, rigorous quasi-experimental evidence specifically addressing its effect on students' conceptual understanding of exponential functions in authentic classroom settings remains limited(Abar & Alencar, 2013)(Nunes et al., 2017).

This gap is particularly significant given the abstract and dynamic nature of exponential functions, which demand coordinated interpretation across multiple representations. Accordingly, there is a need for classroom-based quasi-experimental research that systematically investigates how GeoGebra-based instruction influences high school students' conceptual understanding within real instructional contexts(Dorner & Ableitinger, 2022)(Liburd & Jen, 2021)(Putra et al., 2021). Responding to this need, the present study examines the effect of GeoGebra-based instruction on Grade 10 students' conceptual understanding of exponential

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functions in two Indonesian senior high schools. By employing a quasi-experimental design situated in authentic classroom practice, this research contributes empirically grounded evidence to the discourse on technology-enhanced mathematics instruction and offers contextually situated insights for pedagogical development(Albaladejo et al., 2015)(Hidayat et al., 2023).

2. Method

2.1. Research Design

This study employed a quantitative quasi-experimental design using a nonequivalent control group structure. Two intact classes were assigned as the experimental and control groups. The experimental group received GeoGebra-based instruction, whereas the control group was taught through conventional direct instruction. Students' conceptual understanding of exponential functions was measured using a pretest and posttest administered to both groups. The pretest established baseline comparability, and the posttest assessed the effect of the instructional intervention. This design is appropriate for classroom-based research where random assignment is not feasible, while still allowing systematic comparison between instructional approaches(Seloane et al., 2023)(Yimer & Feza, 2019).

Table 1. Nonequivalent Control Group Design

Group	Pretest (O_1)	Instructional Treatment	Posttest (O_2)
Experimental	O_1	X (GeoGebra-Based Instruction)	O_1
Control	O_2	Conventional Direct Instruction	O_2

Definition,

O_1 = Pretest measuring students' conceptual understanding of exponential functions

X = GeoGebra-based instructional intervention

O_2 = Posttest measuring students' conceptual understanding of exponential functions

2.2. Participants and Data Sources

The study was conducted in two public senior high schools: State Senior High School 2 Palu (*SMA Negeri* 2 *Palu*) and State Senior High School 4 Palu (*SMA Negeri* 4 *Palu*). A total of 132 tenth-grade students were selected through purposive sampling, which included 66 students in the experimental group and 66 in the control group. This technique was used to ensure group homogeneity based on recommendations from mathematics teachers regarding curriculum exposure and instructional settings. The primary data source consisted of students' scores on a validated test measuring conceptual understanding of exponential functions, administered before and after the instructional intervention.

2.3. Research Procedure

The research procedure consisted of structured phases to ensure methodological rigor. During the planning stage, lesson plans were developed for both instructional conditions, GeoGebra-based materials were prepared, and a test measuring students' conceptual understanding of exponential functions was constructed in alignment with the national curriculum. The instrument underwent expert review to establish content validity, followed by empirical testing to

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determine reliability, item difficulty, and discrimination indices.

To control for potential teacher effects, the same mathematics teacher taught both the experimental and control groups. The teacher had prior instructional experience and received preparatory training in implementing GeoGebra-based instruction to ensure consistency and fidelity of treatment across sessions. Following instrument validation, a pretest was administered to both groups to establish baseline comparability in conceptual understanding.

The experimental group then received GeoGebra-based instruction across three 45-minute sessions emphasizing dynamic visualization and multiple representations of exponential functions, whereas the control group received conventional direct instruction covering the same content. After the intervention, a posttest—equivalent in structure and content to the pretest—was administered to measure students' conceptual understanding. Data were subsequently analyzed using descriptive and inferential statistical procedures(Yimer & Feza, 2019).

2.4. Instrumentation and Feasibility Analysis

The primary instrument was a validated test consisting of four essay items designed to assess students' conceptual understanding of exponential functions in alignment with the curriculum's cognitive objectives. Validity was tested using Pearson correlation, and internal consistency was measured via Cronbach's Alpha. Difficulty and discrimination indices were also examined to ensure quality. Table 2 demonstrated an overall reliability (Cronbach's alpha) of 0.71, while the item performance was assessed using Corrected Item-Total Correlation (CITC).

Table 2. Instrument Quality Analysis

Item	r-calculated	r-table	Validity	Mean	Max Score	Difficulty Index	Category	CITC	Discri-mination
1.	0.831	0.219	Valid	17.62	25	0.705	Easy	0.566	Good
2.	0.775	0.219	Valid	21.57	25	0.863	Easy	0.591	Good
3.	0.695	0.219	Valid	22.41	30	0.747	Easy	0.444	Good
4.	0.642	0.219	Valid	7.47	20	0.374	Moderate	0.472	Good

2.5. Data Analysis Techniques

Data analysis was conducted using SPSS software. Descriptive statistics provided summary metrics such as mean and standard deviation. The Kolmogorov–Smirnov test was used to check data normality. Since posttest scores from the control group were not normally distributed ($p < .05$), the Mann–Whitney U test was used to compare the two independent groups (Febrianti & Dasari, 2024)(Uwurukundo et al., 2022). This test is appropriate for non-normal, ordinal, or outlier-prone data. The hypotheses tested were as follows:

a. H_0: There is no significant difference in students' conceptual understanding of exponential functions between those taught using GeoGebra-based instruction and those taught using conventional direct instruction.
b. H_1: There is a significant difference in students' conceptual understanding of exponential functions between those taught using GeoGebra-based instruction and those taught using conventional direct instruction.

2.6. Ethical Considerations

The study adhered to academic ethical standards. Informed consent was obtained from the schools and students involved. Confidentiality of participants' data was

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P-ISSN: 2656-2286

assured, and the intervention posed no risks or undue burden. Instructional activities were aligned with the national curriculum, and equity was maintained across groups. No participant was disadvantaged in the learning process (Mataheru et al., 2023)(Muslim et al., 2023).

3. Results and Discussion

3.1. Overview of Research Implementation

The implementation of the research proceeded as planned, involving active participation from both teachers and students at the two selected senior high schools. Student attendance remained consistently stable throughout the instructional sessions, and no significant technical or administrative issues were encountered. In the experimental group, the integration of GeoGebra stimulated high student engagement, as evidenced by their enthusiasm in exploring the software's features and their active involvement in classroom discussions. Teachers also reported that the media was helpful in visually and dynamically illustrating the concepts of exponential functions.

Table 3. Descriptive Statistics of Pretest and Posttest Scores

Group	N	Pretest Min	Pretest Max	Pretest Mean	Posttest Min	Posttest Max	Posttest Mean
Control	66	36	77	57.41	54	89	81.00
Experimental	66	23	88	60.74	84	93	87.97

In contrast, instruction in the control group was delivered using conventional methods, relying primarily on verbal explanations and written exercises. Although the lessons ran smoothly, the classroom interaction was predominantly one-directional. Assessment for both groups was conducted simultaneously using a pretest and posttest, administered with identical content and within the same time frame to ensure fairness across conditions. Table 3 shows that both groups demonstrated comparable baseline conceptual understanding, as reflected in their pretest means. Following the intervention, the experimental group exhibited a higher posttest mean compared to the control group, indicating a stronger improvement in students' conceptual understanding of exponential functions.

3.2. Statistical Analysis and Interpretation

To ensure the validity of the statistical analysis, it was essential to verify that the data met the required parametric assumptions. Prior to hypothesis testing, normality was examined using the Kolmogorov–Smirnov test. The results are presented in Table 4.

Table 4. Normality Test Results

Data	Sig.
Pretest (Control)	$< .001$
Posttest (Control)	$< .001$
Pretest (Experimental)	$.200$
Posttest (Experimental)	$.063$

Table 4 indicates that the control group data were not normally distributed ($p < .05$). The experimental group satisfied the normality assumption. Accordingly, the non-parametric Mann–Whitney U test was used to compare

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posttest scores between groups.

Table 5. Mann–Whitney U Test Ranks

Group	N	Mean Rank	Sum of Ranks
Experimental	66	83.23	5493.50
Control	66	49.77	3284.50

The higher mean rank of the experimental group suggests stronger post-intervention conceptual understanding compared to the control group. The results in Tables 5 and 6 indicate a statistically significant difference between the experimental and control groups ($p < .001$). The elevated mean rank of the experimental group indicates that the utilization of GeoGebra substantially enhanced students' ability in mastering exponential functions.

Tabel 6. Summary of Mann–Whitney U Test Statistics

Statistic	Value
Mann–Whitney U	1073.50
Wilcoxon W	3284.50
Z	-5.06
p-value (2-tailed)	$< .001$

The Mann–Whitney U test revealed a statistically significant difference in posttest conceptual understanding between the experimental and control groups ($p < .001$). Therefore, the null hypothesis was rejected, indicating that GeoGebra-based instruction produced significantly different outcomes compared to conventional direct instruction. Descriptive statistics further show that the experimental group achieved higher posttest scores, suggesting stronger conceptual understanding of exponential functions following the intervention.

To examine the practical significance of the findings, the effect size (r) was calculated using the formula $r = Z/\sqrt{N}$, where Z represents the standardized test statistic and N refers to the total number of observations across both groups. For non-parametric tests such as the Mann–Whitney U, the total sample size ($N = 132$) is conventionally used in the denominator to estimate effect magnitude (Field, 2018)(Pallant, 2020). With $Z = -5.06$ and $N = 132$, the calculated effect size was,

$$r = \frac{5.06}{\sqrt{132}} \approx 0.44$$

According to commonly accepted benchmarks(Cohen, 1988), this value represents a moderate to large effect, indicating that GeoGebra-based instruction had a practically meaningful impact on students' conceptual understanding of exponential functions, beyond statistical significance. The findings indicate that GeoGebra-based instruction had a moderate-to-large effect ($r = 0.44$) on students' conceptual understanding of exponential functions. Beyond statistical significance, this magnitude suggests meaningful pedagogical impact within authentic classroom settings.

In educational research, effect sizes of this scale are considered substantial, particularly in quasi-experimental designs conducted under natural instructional constraints(Cohen, 1988). The observed effect may be attributed to several

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interrelated cognitive mechanisms activated through dynamic visualization and representational coordination. From a constructivist perspective, GeoGebra facilitates active knowledge construction by enabling students to manipulate parameters and immediately observe changes in graphical representations(Anwar et al., 2025)(Seloane et al., 2025). Such real-time feedback reduces cognitive load associated with abstract symbol manipulation and supports the integration of visual-spatial and symbolic processing. In the context of exponential functions where growth patterns are non-linear and often counterintuitive dynamic visualization helps students internalize structural relationships rather than rely solely on procedural computation.

Drawing on mathematical representation theory, the effect size can also be interpreted as a consequence of strengthened representational fluency. GeoGebra allows simultaneous coordination of algebraic expressions, graphical curves, and numerical tables, fostering deeper structural connections(Mensah & Nabie, 2021)(Safrida et al., 2020). This coordinated multi-representational engagement likely enhanced students' conceptual understanding by clarifying the relationship between exponential parameters and graphical behavior.

Compared with prior research, the observed effect size ($r = 0.44$) lies at the upper range of the moderate category commonly reported in GeoGebra-based interventions(Putra et al., 2021)(Wahyuni et al., 2025). While many studies document small-to-moderate improvements in conceptual understanding, the magnitude found here suggests a comparatively stronger impact within a classroom-based quasi-experimental context. Differences in reported effect sizes may reflect variation in topic specificity, intervention intensity, teacher expertise, and technological readiness(Albaladejo et al., 2015)(Dildabayeva & Zhaydakbayeva, 2024). Given the abstract and non-linear nature of exponential functions, the relatively stronger effect may indicate that dynamic visualization plays a particularly decisive role in supporting conceptual restructuring and reducing misconceptions in this domain(Canonigo, 2024).

Practically, these findings support the structured integration of GeoGebra in teaching conceptually demanding topics at the secondary level. The results provide classroom-based quasi-experimental evidence that dynamic mathematical software can meaningfully strengthen students' conceptual understanding when implemented with instructional fidelity. While the contextualized design enhances ecological validity, the generalizability of the findings remains contingent upon comparable instructional conditions, teacher preparedness, and technological infrastructure(Kul, 2018)(Zagoto et al., 2025)(Zainudin et al., 2025).

Accordingly, several limitations should be acknowledged. The sample was geographically restricted, the design relied exclusively on quantitative measures, and long-term retention of conceptual understanding was not examined. Future research should adopt longitudinal and mixed-method approaches to investigate sustained conceptual development and to determine how varying instructional designs optimize the cognitive affordances of dynamic technology across diverse mathematical domains and student populations.

4. Conclusions

This study demonstrates that GeoGebra-based instruction exerts a statistically significant and practically meaningful effect on students' conceptual understanding of exponential functions. Consistent with the proposed hypothesis,

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a statistically significant difference in conceptual understanding was identified between students who received GeoGebra-based instruction and those who received conventional direct instruction, leading to the rejection of the null hypothesis. The moderate-to-large effect size ($r = 0.44$) indicates that the observed gains extend beyond statistical significance and reflect substantive instructional impact within authentic classroom contexts. The findings reinforce the theoretical proposition that dynamic visualization and coordinated multiple representations facilitate conceptual restructuring in abstract mathematical domains. When implemented with instructional fidelity, GeoGebra supports deeper structural understanding rather than procedural replication, particularly for non-linear concepts such as exponential growth.

Importantly, this research provides empirical quasi-experimental evidence derived directly from authentic classroom implementation in the Indonesian secondary education context, addressing a documented gap concerning rigorous investigations of students' conceptual understanding of exponential functions. As such, the practical implication is grounded not in theoretical projection but in observed instructional practice: structured and fidelity-based integration of GeoGebra can be adopted in comparable classroom settings to strengthen conceptual understanding in conceptually demanding mathematical domains. While generalization should remain context-sensitive, the classroom-based nature of the evidence enhances its pedagogical relevance and applicability. Future studies should extend this line of inquiry by examining long-term retention, diverse learner populations, and variations in instructional design to further clarify the mechanisms through which technology-enhanced environments strengthen conceptual understanding.

Author Contributions

For transparency, we encourage authors to submit an author statement file outlining their individual contributions to the article using the relevant CRediT roles: Conceptualization; Data curation; Formal analysis; Funding acquisition; Investigation; Methodology; Project administration; Resources; Software; Supervision; Validation; Visualization; Roles/Writing - original draft; Writing - review & editing. Authorship statements should be formatted with the names of authors first and CRediT role(s) following.

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Declaration of Competing Interest

Authors must state any potential competing interests that may affect or prejudice their study, such as financial or personal ties, rivalries, or religious convictions. Example of these competing interests include monetary relationships or agreements with an establishment whose invention was utilized or mentioned in the research; any monetary interests derived from a business relationship with a competitor; any remuneration made directly to an author(s) to write the manuscript; and other monetary connections, or other circumstances that may elevate questions about the objectivity of the work or the views expressed within

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the text. Authors may be requested for more information on the nature of the stated interests. The article may be withheld from publishing depending on the specifics of the case. If the manuscript is accepted for publication, the competing interests are disclosed in a note before the references section.

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Optimization of Sandblasting Project Scheduling at PT. Prima Saka Engineering Using the Hungarian Algorithm

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ABSTRACT

This study employs the Hungarian Algorithm to optimize the sandblasting project scheduling at PT. Prima Saka Engineering. Five workers (P1-P5) with different specializations were assigned five tasks: surface preparation (T1), exterior sandblasting (T2), interior sandblasting (T3), primer application (T4), and final quality inspection (T5). The goal was to reduce overall completion time, with each worker being assigned to only one task. The methodical steps comprised matrix building, reduction, and optimality testing. The Hungarian Algorithm generated an optimal assignment with a minimal total time of 1,128 hours, saving 222 hours (16.4%) compared to the previous 1,350-hour method. This study demonstrates that the Hungarian Algorithm provides a mathematically optimal and practically applicable framework for strategic resource allocation and project management.

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1. Introduction

National construction projects play a strategis role in supporting economic growth and infrastructure development. Infrastructure is essential to a country's development, with the ultimate goal being to improve the wellbeing of all of its residents (Arindi et al., 2025). However, the construction industry frequently faces a number of issues, such as schedule delays and issues with material availability. These challenges necessitate a more precise and adaptable method of project management and planning.

Mathematical methods are becoming more and more significant in this environment. Mathematical concepts like computing, assessment, and statistical

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analysis are crucial in business, management, and finance because they facilitate forecasting, lower uncertainty, and allow decision-making based on reliable data and analysis (Atmaja, 2024). Optimization is one branch of mathematics that makes a substantial contribution to operational efficiency, especially when it comes to assignment difficulties. The assignment issue is a particular instance of linear programming that focuses on finding the most effective way to distribute few resources across a number of jobs (Rahmawati et al., 2015). Finding the best distribution strategy is the main goal, which may be achieved by either increasing overall revenue or reducing overall expenses or time (Monica et al., 2025).

The Hungarian Algorithm is among the most popular and effective approaches for resolving assignment problems among the many others. By solving assignment difficulties, the Hungarian Algorithm is a powerful optimization method that helps companies boost income (Basriati & Lestari, 2017). In order to minimize overall expenses or time, this algorithm can identify the best ways to distribute resources among activities. The Hungarian Algorithm saves a significant amount of time and yields the best outcomes in many industrial planning scenarios (Yulistiana et al., 2015). It is computationally efficient and dependable for moderately sized problems because of its combinatorial optimization approach, which ensures an optimal solution in polynomial time.

The use of the Hungarian Algorithm for sandblasting project scheduling should produce the best way to match work teams with specific projects. This approach aims to reduce operating costs, increase resource utilization, and reduce the overall time needed to complete all tasks (Pratama et al., 2020). Since the Hungarian Algorithm does not need the formulation of objective functions and constraints in standard LP form, it is more precise and efficient for assignment problems than other optimization approaches such as the generic linear programming example simplex method. Metaheuristic methods such as evolutionary algorithms or simulated annealing can handle larger and more complex problems, but they often require longer computing times and parameter tuning, and they do not offer global optimality. The Hungarian Algorithm, on the other hand, provides a precise and reliable solution for the balanced assignment problem, which is particularly helpful when the tasks and resources are clearly defined, as in project scheduling.

Despite its advantages, the Hungarian Algorithm has drawbacks. It is designed to operate with balanced problems when the number of workers and jobs is equal. If the problem is unbalanced, dummy variables must be added, which may restrict its use in extremely unpredictable situations by influencing accuracy or the uncertainty of work length.

Schedule optimization is essential for PT. Prima Saka Engineering to ensure the efficient use of skilled workers in sandblasting projects. Five people with various skills must be assigned by the company to five projects, each with its own set of technical requirements and timeframes. Lack of a systematic assignment process may lead to inefficiencies, including misaligned worker task allocation, extended project durations, and increased operating costs.

The application of the Hungarian Algorithm to the scheduling of sandblasting projects is yet relatively unexplored, despite its widespread use in manufacturing, logistics, and educational scheduling. As far as the authors are aware, no previous study has specifically examined how PT. Prima Saka Engineering optimized the assignment of sandblasting tasks using the Hungarian Algorithm. This study aims

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to bridge that gap by applying the technique to shorten the total project completion time while accounting for worker specialization and task characteristics. The results are expected to provide the company with a quantitative and systematic scheduling framework that supports strategic resource allocation and project management decision-making.

2. Method

An operations research strategy was employed to optimize the scheduling of sandblasting projects using the Hungarian method. The investigation was carried out by PT. Prima Saka Engineering, a company that provides surface preparation and coating services. The primary focus of this study is assigning workers to project tasks in order to shorten the total completion time.

2.1. Data Analysis

The data used in this study includes five project jobs with distinctive qualities and five people with a range of specializations. Surface preparation (P3), painting and coating (P4), manual sandblasting (P1), automatic sandblasting (P2), and quality control (P5) are the five groups into which the staff members are separated based on their areas of competence. The project's tasks include primer application (T4), interior and exterior sandblasting (T3), surface preparation (T1), and final quality inspection (T5).

2.1.1. Data Type and Source

The kind of secondary data used are estimates of how long it takes each employee to complete each task. These estimations were derived from the company's project papers and historical records. The choice to employ secondary data was influenced by a number of reasons.:

1. Time Efficiency: Primary data collection through direct observation or experimentation would require extended monitoring of ongoing sandblasting projects, which could take several months to complete. Given the time constraints of this research, secondary data provides a more feasible alternative.
2. Access Limitations: PT. Prima Saka Engineering operates on a project basis with varying schedules. Coordinating direct observations for all five workers across all five task types would require significant coordination and may disrupt ongoing operations.
3. Cost Considerations: Primary data collection would involve additional costs for equipment, monitoring, and personnel. The use of existing company records eliminates these expenses while still providing reliable estimates.
4. Data Availability: The company maintains comprehensive project records, including time estimates based on historical performance. These estimates have been validated through multiple project cycles and reflect realistic working times.

2.1.2. Data Structure

The data is arranged as an assignment matrix, where workers (P1–P5) are represented by rows and tasks (T1–T5) by columns. Each cell in the matrix contains the estimated time (in hours) required for a specific worker to complete a particular task. This matrix serves as the Hungarian Algorithm's input.

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2.2. Determining the Assignment Problem

Assignment issues are found using the following methods: Create a 5x5 assignment matrix that shows the amount of time spent on each task. To reduce a row, subtract each element from the lowest row value; to reduce a column, subtract each element from the least column value; and to cover all zero items with the fewest lines possible, either vertically or horizontally. (5) confirming optimality by comparing the number of lines with the matrix's order; (6) improving the values by subtracting the smallest value from the components not covered by the lines and adding it to the components at the line intersection; (7) calculating the shortest total time and choosing the best assignment from the available zero elements; and (6) repeating step (4) until the best choice is discovered.

2.3. Determining the Hungarian Algorithm

The Hungarian algorithm approach looks for the optimal worker-task allocation by cutting overall costs and time. The constraint function is then obtained and included in the assignment table,

$x_{ij} = 1$ if worker i is assigned to 1 of the jobs
$x_{ij} = 0$ if worker i is not assigned to any job.

Mathematical model for equation 1 assignment problems(Azmi & Kartika, 2025),

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij} \quad (1)$$

with obstacles,

$$\sum_{i=1}^{m} x_{ij} = 1 \; ; j = 1, 2, \ldots, m \quad (2)$$

$$\sum_{j=1}^{n} c_{ij} = 1 \; ; i = 1, 2, \ldots, n \quad (3)$$

Description,
Z : Objective function for which the optimal value is sought
n : Number of tasks to be completed
x_{ij} : Assignment from i to j
C_{ij} : Allocation parameter from i to j
i : employee i
j : Tasks or job j

2.4. Project Scheduling

Scheduling includes deciding when a work should be finished and assigning resources, like personnel, tools, or space, for certain operations (Sallaby & Kanedi, 2020). Making the most of the available resources to get the greatest results is the main objective of scheduling (Nufus, 2024). Project scheduling is the process of planning to complete a project on time and within the budget allotted. (Nanda et al., 2023).

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P-ISSN: 2656-2286

2.5. Assignment Problem

Assignment problems are a specific situation in linear programming, which concentrates on distributing resources across several jobs (Rahmawati et al., 2015). The primary objective of this task is to determine the best distribution plan, either by increasing overall revenue or decreasing overall expenses (Monica et al., 2025).

2.6. Sandblasting

Sandblasting involves blowing a stream of abrasive particles, usually made of silica sand or steel grit, at high speed onto a surface. The goal of this technique is to get rid of surface contaminants like rust, paint, and oil (Purnamayasa, 2024). This process serves to remove rust and corrosion, clean old paint, create a surface profile for coating, and prepare the surface before painting. Many issues come up during the sandblasting process, such as damaged water inverters, broken nozzle hoses, and machines with reduced speed loss, which reduces effectiveness (Kurniawan et al., 2024).

2.7. Project Management

According to George R. Terry, management is a specific process that involves planning, organizing, and deploying human resources along with other resources in order to achieve predefined objectives (Asnuddin et al., 2018). Project management, on the other hand, is the application of planning, organizing, directing, and controlling resources to achieve specific goals within predetermined time, cost, and quality constraints.

2.8. Optimization

The objective of the computer work known as optimization is to identify the optimal solution from a set of potential solutions while adhering to many predetermined constraints (Devita & Wibawa, 2020). The optimization involves utilizing designated algorithms to enhance efficiency or minimize operational expenses required to accomplish the specific project.

3. Results and Discussion

This section displays the results of PT. Prima Saka Engineering's sandblasting project scheduling optimization utilizing the Hungarian Algorithm. Five project tasks and the anticipated working times for five personnel with different specialties were included in the secondary data from corporate project reports that were used for the research.

3.1. Problem Formulation

The following is how the assignment problem for this study was constructed. There is only one worker allocated to each task,

$$\sum_{j=1}^{5} x_{ij} = 1,\ i = 1, 2, \ldots., 5 \tag{4}$$

There is only one worker allocated to each task,

$$\sum_{i=1}^{5} c_{ij} = 1\ ; j = 1, 2, \ldots., 5 \tag{5}$$

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P-ISSN: 2656-2286

Constraint in binary,

$$x_{ij} \in (0,1), \forall i \quad (6)$$

3.2. Data Presentation

Based on company data, the estimated working times for each worker on each task are presented in **Table 1.** This table lists the codes and specializations of five employees who worked on a sandblasting project at PT. Prima Saka Engineering. Worker A (P1) is an expert in manual sandblasting, which entails using sandblasting equipment by hand and calls for specific abrasive jet direction skills.

Table 1. Worker Data

Name	Specialization	Worker Code
A	Manual Sandblasting	P1
B	Automatic Sandblasting	P2
C	Surface Preparation	P3
D	Painting and Coating	P4
E	Quality Control	P5

Using an automated system that may boost productivity over wide areas, Worker B (P2) specializes in automatic sandblasting. Worker C (P3) is an expert in surface preparation, which entails getting surfaces ready for sandblasting. After sandblasting, Worker D (P4) applies priming and protective coatings as part of his painting and coating expertise. In the meantime, Worker E (P5) is in charge of quality control, making sure that output satisfies predetermined criteria. Since each worker has unique skills and processing times for every kind of task, these specialized differences are the main factors taken into account while generating the assignment matrix.

Table 2. Assignment Data

Job Description	Area (m^2)	*Deadline (Hours)*	Task Code
Surface Preparasi (griding)	500	30	T1
Exterior Sandblasting Area	300	25	T2
Interior Sandblasting Area	400	35	T3
Primer Application	350	28	T4
Final Quality Inspection	200	15	T5

The location, deadline, and task code for each of the five primary categories of work in the sandblasting process series are shown in Table 2's project assignment data. The largest work that helps to prepare the surface prior to the sandblasting process is Task T1, Surface Preparation (grinding), which spans an area of 500 m^2 and has a 30-hour deadline. Task T2, Exterior Sandblasting Area, has a 25-hour deadline and spans a 300-square-meter area. Its goal is to remove dirt, rust, and old layers from exterior surfaces.

Due to the intricacy of the work in the interior region, Task T3, Interior Sandblasting region, has the longest time allotment. It has a 400 m^2 area and a 35-hour deadline. Task T4, Primer Application, has a 28-hour deadline and covers an area of 350 m^2. It entails applying a primer layer as a basic protection following sandblasting. Task T5, Final Quality Inspection, is the last inspection step to make

E-ISSN: 2656-5846
P-ISSN: 2656-2286

sure that all work satisfies the necessary quality standards. It spans an area of 200 m² and has a 15-hour deadline. This information serves as the foundation for creating the assignment matrix, which serves as the primary input for the Hungarian Algorithm.

Table 3. Work Time Data (in hours)

Worker	Assignment				
	T1	**T2**	**T3**	**T4**	**T5**
P1	240	360	300	420	185
P2	270	243	428	480	300
P3	210	420	270	360	330
P4	360	480	450	210	460
P5	370	330	345	320	165

The main input for the Hungarian Algorithm's assignment matrix generation is the estimated processing time (in hours) for each worker and job combination, which is shown in Table 3. This information, which comes from the company's past project records, shows each employee's true capacity to finish every kind of task. Task T5 (Final Quality Inspection) took worker P1 (Manual Sandblasting) the least amount of time (185 hours), however task T4 (Primer Application) took the longest (420 hours). At 243 hours, worker P2 (Automatic Sandblasting) completed task T2 (Exterior Sandblasting) with the highest efficiency, which is consistent with his expertise in running automated machinery.

At under 210 hours, Worker P3 (Surface Preparation) performed exceptionally well on task T1 (Surface Preparation/Grinding), demonstrating the fit between his area of expertise and the nature of the work. Given his proficiency in applying protective coatings, worker P4 (Painting and Coating) completed job T4 (Primer Application) in the minimum amount of time (210 hours). In the meantime, Worker P5 (Quality Control) finished T5 (Final Quality Inspection) the quickest, taking 165 hours—the shortest time in the whole matrix. In order to systematically reduce the project's overall completion time, assignment optimization using the Hungarian Algorithm is required, as Table 3 generally demonstrates that there is considerable difference in time amongst workers for the same activity.

3.3. Implementation of the Hungarian Algorithm

The Hungarian Algorithm was implemented through the following sequential steps:

Step 1: Create the Assignment Matrix

The working time data was simplified by dividing all values by 10 to facilitate calculations, resulting in the reduced assignment matrix shown in **Table 4**.

Table 4. Reduced assignment matrix

Worker	Assignment				
	T1	**T2**	**T3**	**T4**	**T5**
P1	24	36	30	42	18,5
P2	27	24,3	42,8	48	30
P3	21	42	27	36	33
P4	36	48	45	21	46
P5	37	33	34,5	32	16,5

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Step 2: Row Reduction
For each row, the smallest value was identified and subtracted from all elements in that row.

Table 5. Row Reduction Results

Worker	Assignment				
	T1	T2	T3	T4	T5
P1	5,5	17,5	11,5	23,5	0
P2	2,7	0	18,5	23,7	5,7
P3	0	21	6	15	12
P4	15	27	24	0	25
P5	20,5	16,5	18	15,5	0

Step 3: Column Reduction
After row reduction, column T3 had no zero value. The smallest value in column T3 was 6.0, which was subtracted from all elements in that column.

Table 6. Column Reduction Results

Worker	Assignment				
	T1	T2	T3	T4	T5
P1	5,5	17,5	5,5	23,5	0
P2	2,7	0	12,5	23,7	5,7
P3	0	21	0	15	12
P4	15	27	18	0	25
P5	20,5	16,5	12	15,5	0

Step 4: Line Drawing for Zero Coverage
The minimum number of lines required to cover all zero elements was drawn. Since the number of lines (4) was less than the matrix order (5), the solution was not yet optimal.

Table 7. Lines Drawing Results (First Iteration)

Worker	Assignment				
	T1	T2	T3	T4	T5
P1	5,5	17,5	5,5	23,5	0
P2	~~2,7~~	~~0~~	~~12,5~~	~~23,7~~	~~5,7~~
P3	~~0~~	~~21~~	~~0~~	~~15~~	~~12~~
P4	~~15~~	~~27~~	~~18~~	~~0~~	~~25~~
P5	20,5	16,5	12	15,5	0

Step 5: Matrix Improvement
Subtracted from all uncovered elements, added to elements at line intersections

Table 8. Improved Matrix

Worker	Assignment				
	T1	T2	T3	T4	T5
P1	~~0~~	~~12~~	~~0~~	~~18~~	~~0~~
P2	2,7	0	12,5	23,7	11,2
P3	~~0~~	~~23,7~~	~~0~~	~~15~~	~~17,5~~
P4	~~15~~	~~29,7~~	~~18~~	~~0~~	~~30,5~~

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P5	~~15~~	~~11~~	~~6,5~~	~~10~~	~~0~~

Step 6: Optimality Check
Five lines were required to cover all zeros, which equals the matrix order, indicating that an optimal solution had been achieved.

Table 9. Optimal assignment results

Worker	Assignment				
	T1	**T2**	**T3**	**T4**	**T5**
P1	0	12	0	18	0
P2	2,7	0	12,5	23,7	11,2
P3	0	23,7	0	15	17,5
P4	15	29,7	18	0	30,5
P5	15	11	6,5	10	0

Based on **Table 9**, the optimal minimum time is obtained, namely:

$$Z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij} \tag{7}$$

$$Z = 300 + 243 + 210 + 210 + 65$$
$$Z = 1.128$$

3.4. Interpretation of Results
The optimal assignment yields the following interpretations:
1. The findings of the optimal assignment indicate that, given a 300-hour working time, manual sandblasting (P1) is the most effective method for completing the interior sandblasting task (T3).
2. The results of the optimal assignment indicate that, with a working time of 243 hours, automatic sandblasting (P2) is the most effective method for completing the outside sandblasting task (T2).
3. The findings of the optimal assignment indicate that, with a working time of 210 hours, surface preparation (P3) is the most effective method for completing the surface preparation (grinding) task (T1).
4. According to the results of the ideal assignment, painting and coating (P4) is the best option for completing the primer application task (T4) in 210 hours.
5. According to the results of the optimal assignment, quality control (P5) is best suited to complete the sandblasting control area task in 165 hours.

This interpretation allows PT. Prima Saka Engineering to determine how the optimization outcomes of the Hungarian Algorithm technique produce systematic and measurable scheduling solutions.

4. Conclusions
This study successfully optimized the scheduling of sandblasting projects at PT. Prima Saka Engineering by applying the Hungarian Algorithm to the assignment problem involving five personnel with different specializations and five project assignments. P1 (Manual Sandblasting) was assigned to T3 (Interior Sandblasting) for 300 hours, P2 (Automatic Sandblasting) to T2 (Exterior Sandblasting) for 243 hours, P3 (Surface Preparation) to T1 (Surface Preparation) for 210 hours, P4 (Painting and Coating) to T4 (Primer Application) for 165 hours, and P5 (Quality

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Control) to T5 (Final Quality Inspection) in order to reach a minimum total completion time of 1,128 hours.

Compared to the company's previous assignment approach, which, based on previous project data, needed around 1,350 hours, the Hungarian Algorithm saved 222 hours, or 16.4%, of time, suggesting a significant boost in efficiency. These findings provide PT. Prima Saka Engineering with a systematic and quantitative scheduling technique that can support strategic resource allocation and decision-making in project management. Future research could look at how the Hungarian Algorithm can be combined with other optimization strategies or used to more complex scenarios with additional restrictions like resource availability, budgetary limitations, or uncertainty factors.

Author Contributions

The first author was tasked with the collection, processing, and analysis of the data, in addition to composing the scientific article. The second author provided criticism and recommendations to enhance the study, its analysis, and the manuscript as a whole.

Declaration of Competing Interest

The authors have declared no indications of potential conflicts of interest.

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